CSCI567 Machine Learning (Spring 2021)

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University of Southern California

March 5, 2021

Outline

1. Logistics

2. Decision trees

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Logistics

- HW 3 is due today.
- Solutions for Quiz 1 were released.

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We have seen different ML models for classification/regression:

- linear models, neural nets and other nonlinear models induced by kernels

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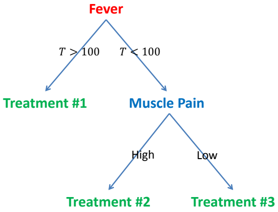
- nonlinear in general
- works for both classification and regression; we focus on classification
- one key advantage is good interpretability
- used to be very popular; ensemble of trees (i.e. "forest") can still be very effective

Example

Many decisions are made based on some tree structure

Medical treatment



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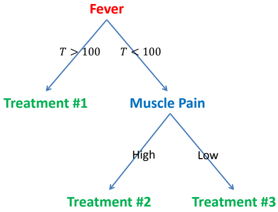
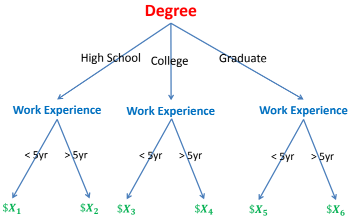
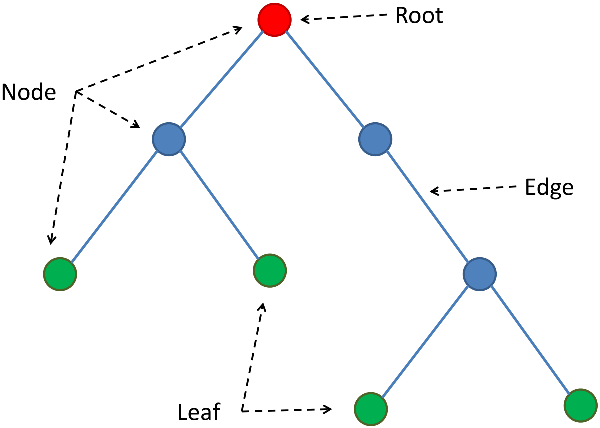


Salary in a company



Tree terminology



A more abstract example of decision trees

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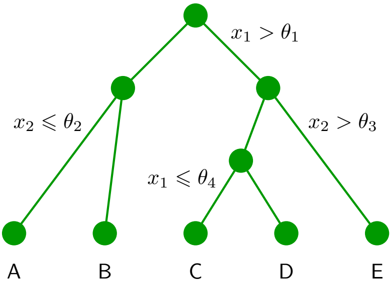

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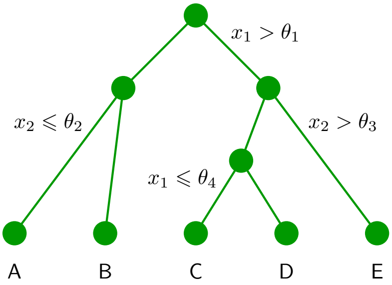

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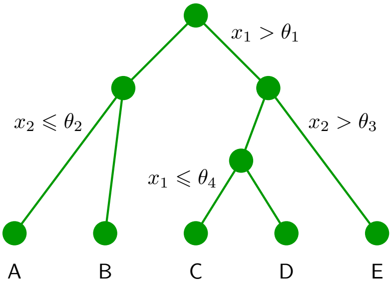

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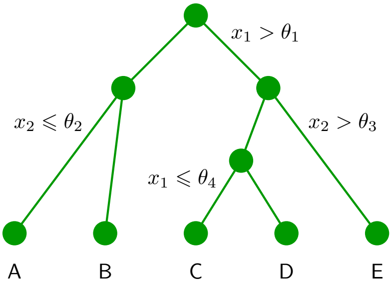

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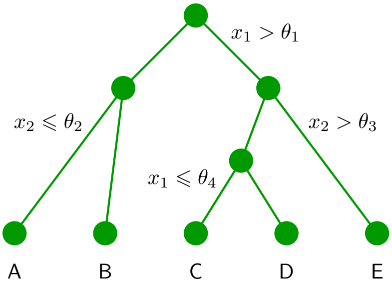

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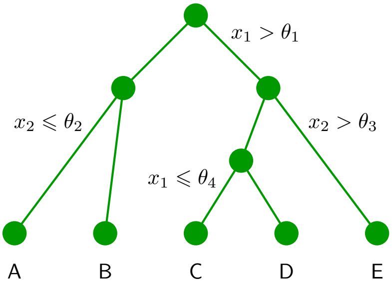

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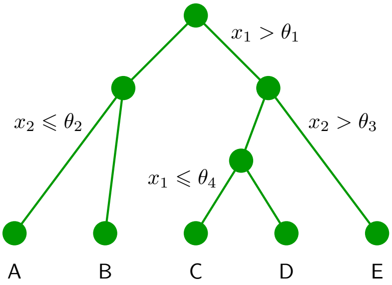

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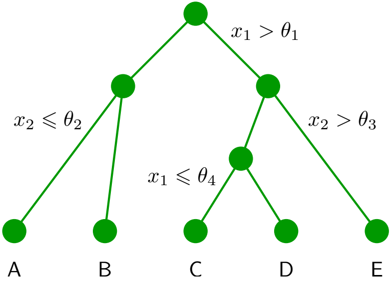
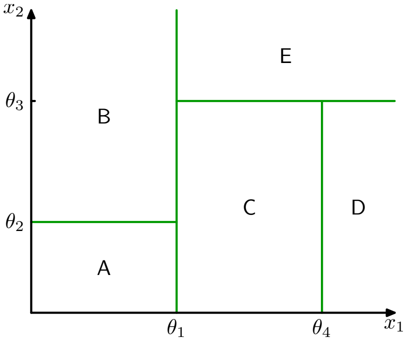
Complex to formally write down, but easy to represent pictorially or as codes.

The decision boundary

Corresponds to a classifier with boundaries:





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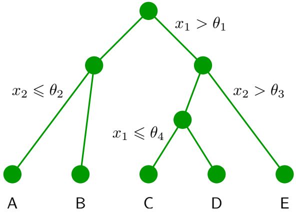

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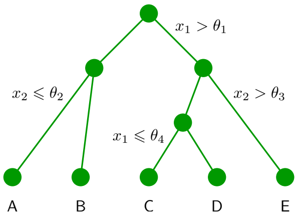

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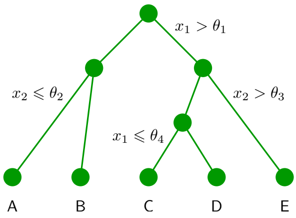

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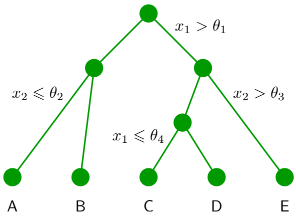

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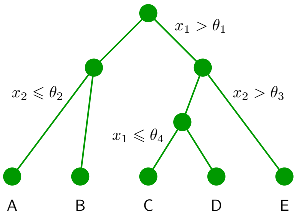

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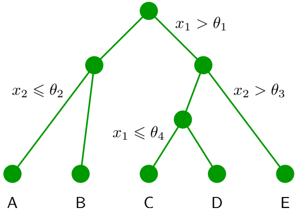

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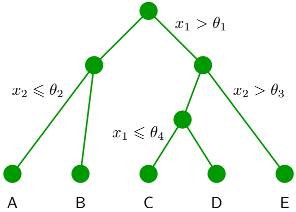

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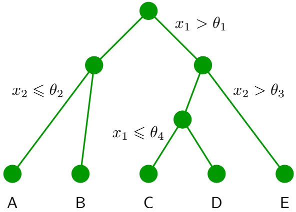

- the value/prediction of the leaves (A, B, . . .)

Learning the parameters

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Instead, we turn to some **greedy top-down approach**.

A running example

[Russell & Norvig, AIMA]

- predict whether a customer will wait for a table at a restaurant
- 12 training examples
- 10 features (all discrete)

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

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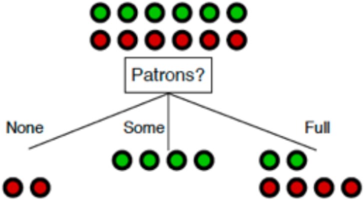
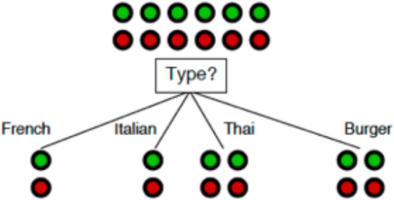
I.e., which feature should we test at the root? Examples:





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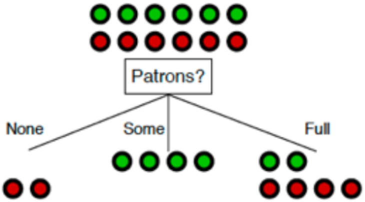

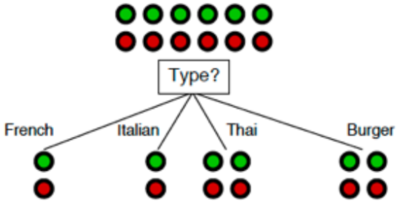

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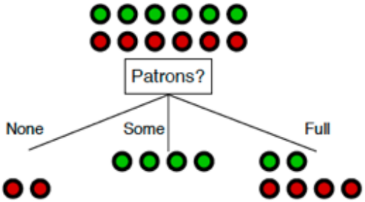

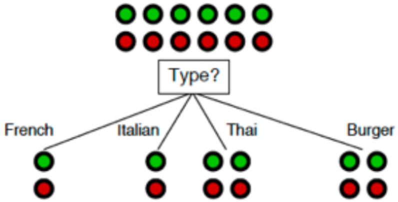

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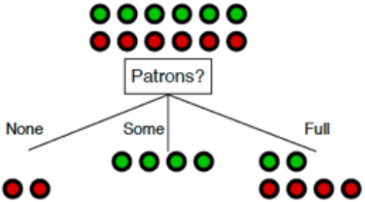

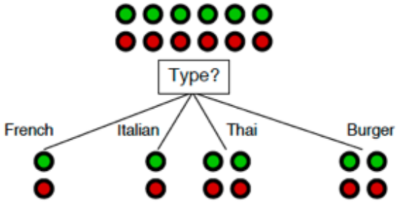

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- how to quantify this intuition?

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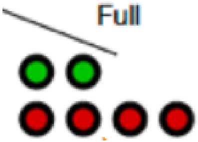

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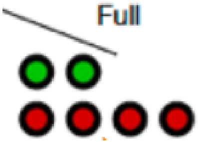

One classic uncertainty measure of a distribution is its *(Shannon) entropy*:

$$H(Y) = -\sum_{k=1}^{\mathsf{C}} P(Y = k) \log P(Y = k)$$

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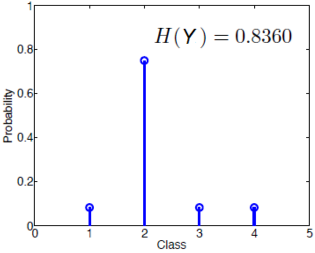
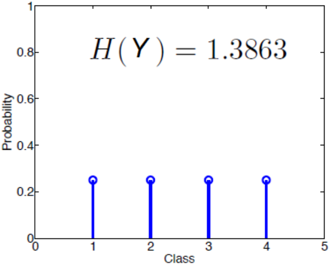
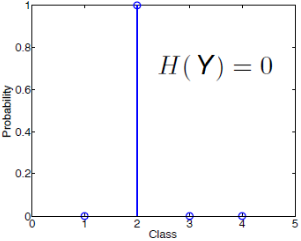
  - e.g. $P = (1, 0, \ldots, 0)$
  - $0 \log 0$ is defined naturally as $\lim_{z \to 0+} z \log z = 0$

Examples of computing entropy

With base e and 4 classes:







Another example

Entropy in each child if root tests on "patrons"

For "None" branch

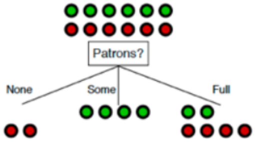
$$-\left(\frac{0}{0+2}\log\frac{0}{0+2}+\frac{2}{0+2}\log\frac{2}{0+2}\right)=0$$

For "Some" branch

$$-\left(\frac{4}{4+0}\log\frac{4}{4+0}+\frac{0}{4+0}\log\frac{0}{4+0}\right)=0$$

For "Full" branch

$$-\left(\frac{2}{2+4}\log\frac{2}{2+4}+\frac{4}{2+4}\log\frac{4}{2+4}\right)\approx 0.9$$



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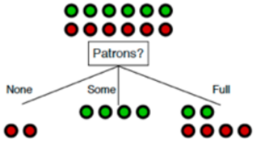

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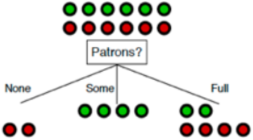

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Very naturally, we take the weighted average of entropy:

$$\frac{2}{12}\times 0+\frac{4}{12}\times 0+\frac{6}{12}\times 0.9=0.45$$

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$$= \sum_{a} P(A = a) H(Y \mid A = a)$$

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$$
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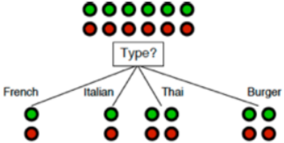
Pick the feature that leads to the smallest conditional entropy.

Deciding the root



For "French" branch

$$-\left(\frac{1}{1+1}\log\frac{1}{1+1}+\frac{1}{1+1}\log\frac{1}{1+1}\right)=1$$

For "Italian" branch

$$-\left(\frac{1}{1+1}\log\frac{1}{1+1}+\frac{1}{1+1}\log\frac{1}{1+1}\right)=1$$

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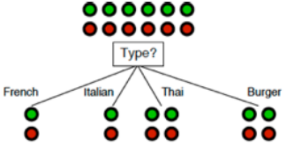

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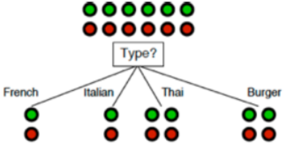

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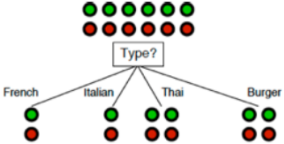

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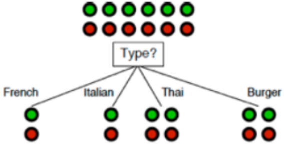

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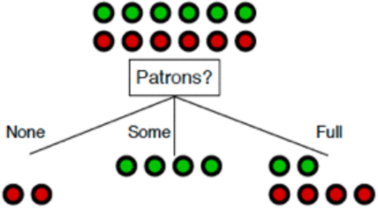
So splitting with "patrons" is better than splitting with "type".

In fact by similar calculation "patrons" is the best split among all features.

We are now done with building the root (this is also called a **stump**).

Repeat recursively

Split each child in the same way.



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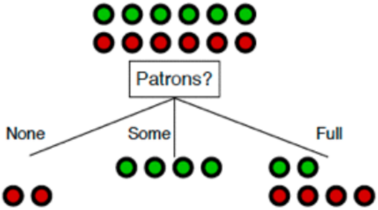

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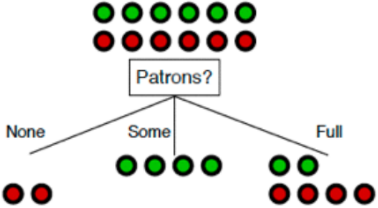
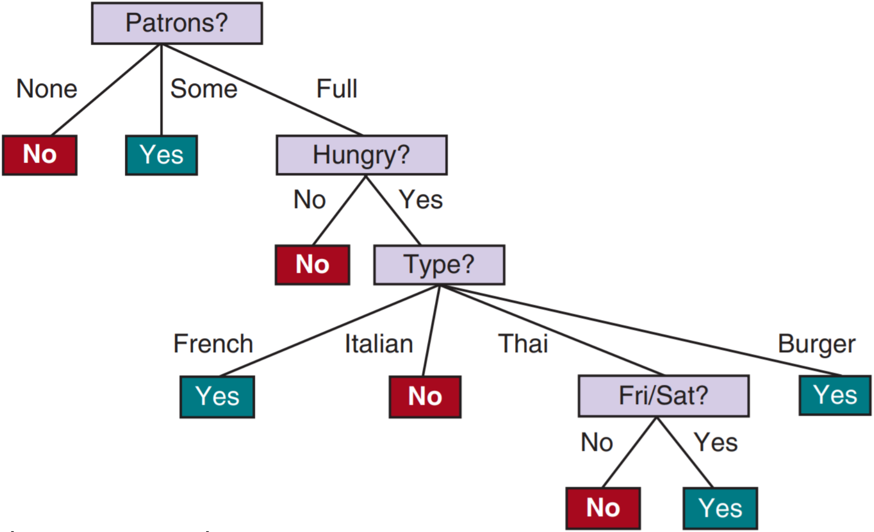
- but no need to split children "none" and "some": they are pure already and become leaves
- for "full", repeat, focusing on those 6 examples:



	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T



Again, very easy to interpret.

Putting it together

DecisionTreeLearning(Examples, Features)

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find the best feature A to split (e.g. based on conditional entropy)

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- if Examples have the same class, return a leaf with this class

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Putting it together

DecisionTreeLearning(Examples, Features)

- if Examples have the same class, return a leaf with this class
- else if Features is empty, return a leaf with the majority class
- else if Examples is empty, return a leaf with majority class of parent
- else

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Variants

Popular decision tree algorithms (e.g. C4.5, CART, etc) are all based on this framework.

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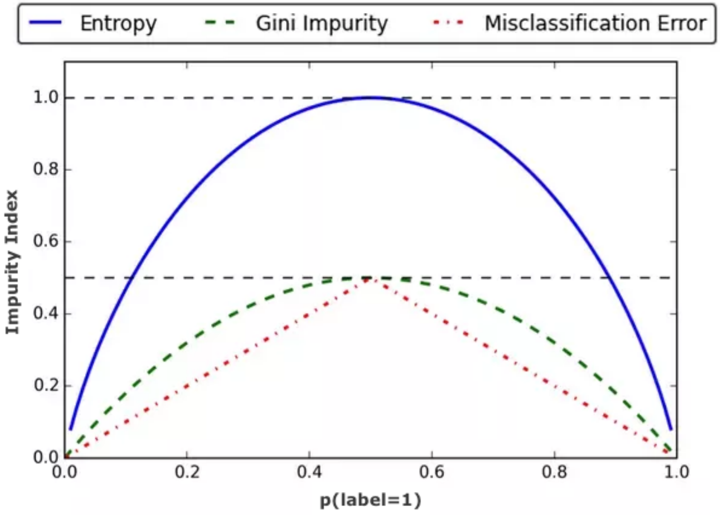
- if a feature is continuous, we need to find a threshold that leads to minimum conditional entropy or Gini impurity.

Variants



1

Image Credit: `https://medium.com/@jason9389/gini-impurity-and-entropy-16116e754b27`

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- all make use of a validation set