CSCI567 Machine Learning (Spring 2021)

Sirisha Rambhatla

University of Southern California

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Outline

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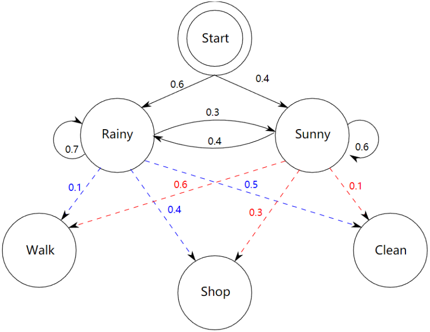
Hidden Markov Models

Model parameters:

- **initial distribution**
  $P(Z_1 = s) = \pi_s$
- **transition distribution**
  $P(Z_{t+1} = s' \mid Z_t = s) = a_{s,s'}$
- **emission distribution**
  $P(X_t = o \mid Z_t = s) = b_{s,o}$



Baum–Welch algorithm

Step 0 Initialize the parameters $(\boldsymbol{\pi}, \boldsymbol{A}, \boldsymbol{B})$

Step 1 (E-Step) Fixing the parameters, compute forward and backward messages for all sample sequences, then use these to compute $\gamma_s^{(n)}(t)$ and $\xi_{s,s'}^{(n)}(t)$ for each n, t, s, s'.

Step 2 (M-Step) Update parameters:

$$\pi_s \propto \sum_n \gamma_s^{(n)}(1), \quad a_{s,s'} \propto \sum_n \sum_{t=1}^{T-1} \xi_{s,s'}^{(n)}(t), \quad b_{s,o} \propto \sum_n \sum_{t:x_t=o} \gamma_s^{(n)}(t)$$

Step 3 Return to Step 1 if not converged

Viterbi Algorithm

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For each $s \in [S]$, compute $\delta_s(1) = \pi_s b_{s,x_1}$.

For each $t = 2, \ldots, T$,

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$$\delta_s(t) = b_{s,x_t} \max_{s'} a_{s',s} \delta_{s'}(t-1)$$

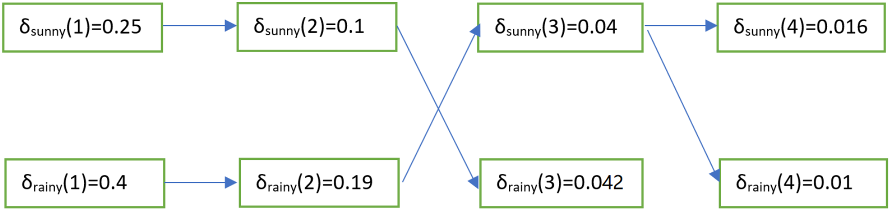
$$\Delta_s(t) = \operatorname*{argmax}_{s'} a_{s',s} \delta_{s'}(t-1)$$

Backtracking: let $z_T^* = \operatorname{argmax}_s \delta_s(T)$.
For each $t = T, \ldots, 2$: set $z_{t-1}^* = \Delta_{z_t^*}(t)$.

Output the most likely path $z_1^*, \ldots, z_T^*$.

Example

Arrows represent the "argmax", i.e. $\Delta_s(t)$.



The most likely path is **"rainy, rainy, sunny, sunny"**.

Viterbi Algorithm with missing data

Viterbi Algorithm with partial data $x_{1:T_0}$

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$$\delta_s(t) = \begin{cases} b_{s,x_t} \max_{s'} a_{s',s}\delta_{s'}(t-1) & \text{if } t \le T_0 \\ \max_{s'} a_{s',s}\delta_{s'}(t-1) & \text{else} \end{cases}$$

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Broadly, these are called **online decision making problems**.

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- make another move...

Two formal setups

We discuss two such problems:

- **multi-armed bandit** (this lecture)
- **reinforcement learning** (next lecture)

Mulit-armed bandits: motivation

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- like a bandit with multiple arms (hence the name)
- **if I can play for 10 times, which machines should I play?**

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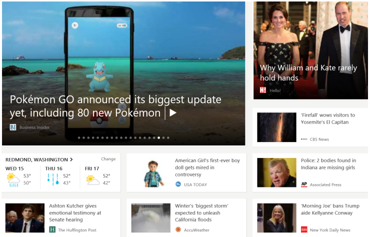

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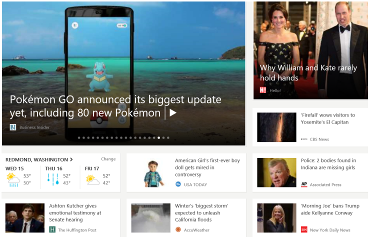

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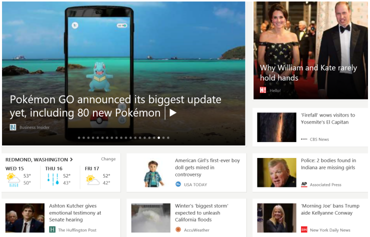
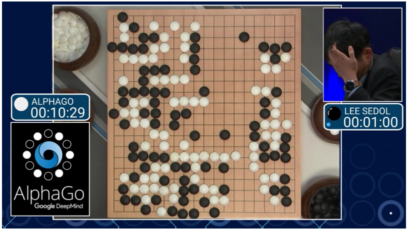
- game playing, each possible move is an arm (AlphaGo indeed has a bandit algorithm as one of the components)





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This kind of limited feedback is now usually referred to as bandit feedback

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This is called the **regret**: *how much I regret for not sticking with the best fixed arm in hindsight?*

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We focus on a simple setting:

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- each arm has a different mean $(\mu_1, \ldots, \mu_K)$; the problem is essentially about finding the best arm $\mathrm{argmax}_a\, \mu_a$ as quickly as possible

Empirical means

Let $\hat{\mu}_{t,a}$ be the **empirical mean** of arm a up to time t:

$$\hat{\mu}_{t,a} = \frac{1}{n_{t,a}} \sum_{\tau \leq t : a_\tau = a} r_{\tau,a}$$

where

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Concentration: $\hat{\mu}_{t,a}$ should be close to μ_a if $n_{t,a}$ is large

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- the algorithm will never pick arm 1 again!

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We next discuss **three ways** to trade off exploration and exploitation for our simple multi-armed bandit setting.

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Parameter T_0 clearly controls the exploration/exploitation trade-off

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- clearly it won't work if the environment is changing

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Is there a *more adaptive* way to explore?

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- a parameter-free algorithm, and *it enjoys optimal regret!*

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Another way to interpret UCB, "**optimism in face of uncertainty**":

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This principle is useful for many other bandit problems.