

MASTER'S THESIS ON

SEMI-BLIND SOURCE SEPARATION VIA
SPARSE REPRESENTATIONS AND ONLINE
DICTIONARY LEARNING

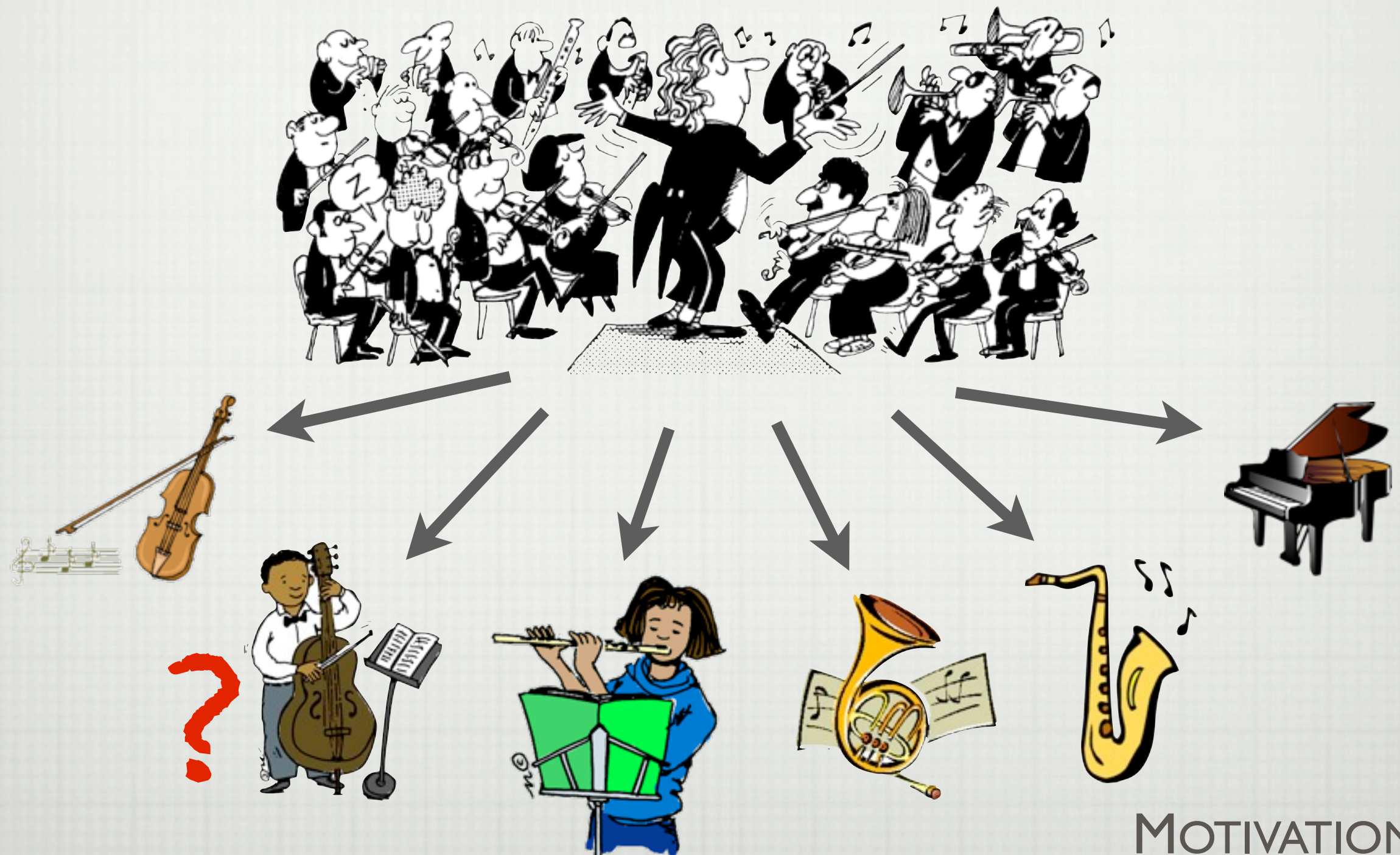
BY

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ADVISOR: PROF. JARVIS HAUPT

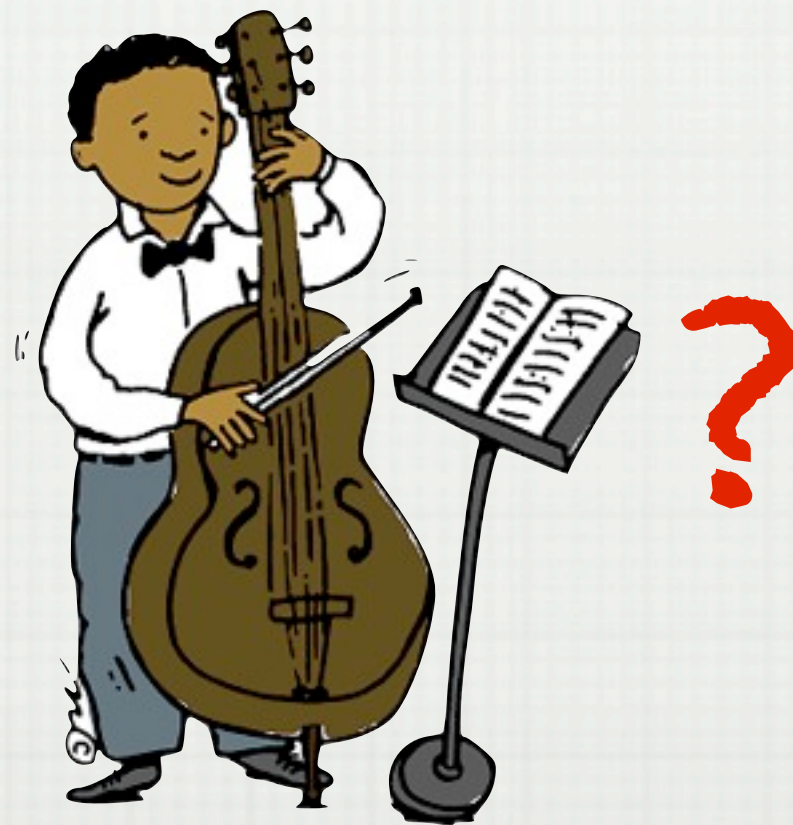


THE LAST TIME I WAS AT THE MINNESOTA ORCHESTRA..



MOTIVATION

How Do I IDENTIFY WHAT CELLO SOUNDS LIKE?



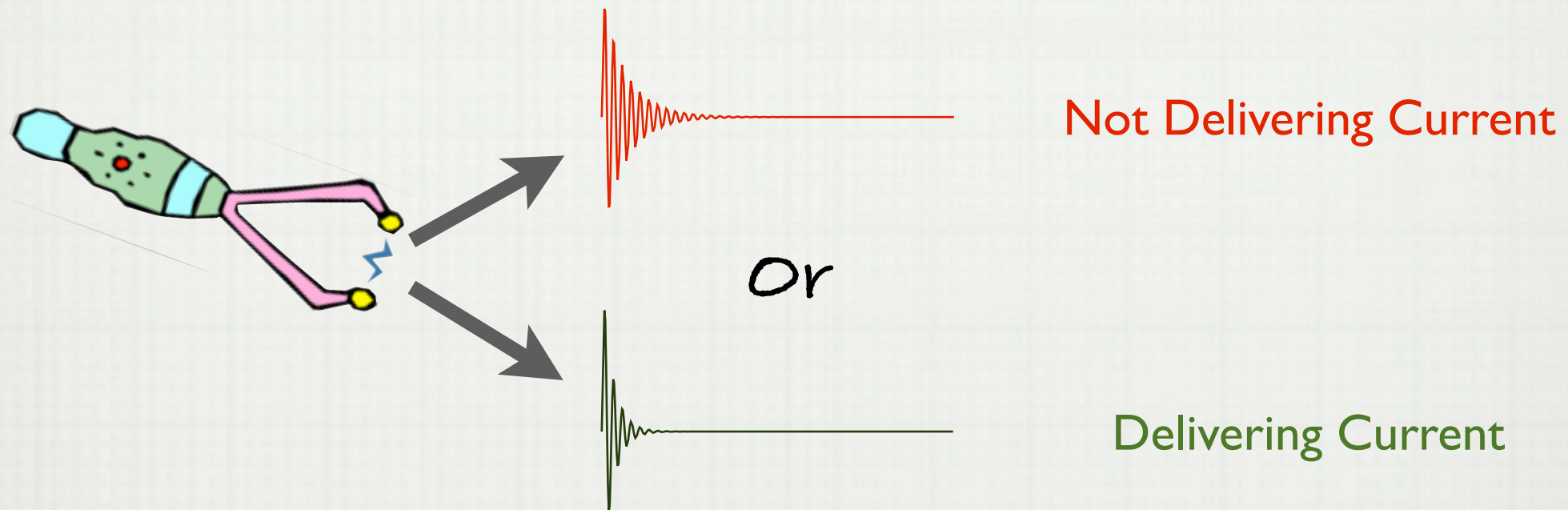
MOTIVATION

THE COCKTAIL PARTY PROBLEM : BLIND SOURCE SEPARATION



- ☐ Multiple speakers are simultaneously speaking
- ☐ The aim is to separately comprehend each speaker.
- ☐ None of the sources are known a-priori - ***Blind Source Separation Problem***

CONSIDER AN AUDIO FORENSICS APPLICATION



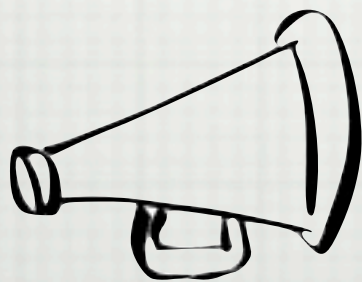
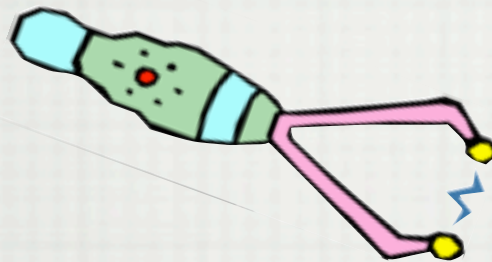
- ☐ Electro-shock Law enforcement devices generate characteristic **Nominally Periodic** signals, indicating discharge of current.
- ☐ These signals are often corrupted by background noise like speech, etc., not known *a-priori*.

MOTIVATION

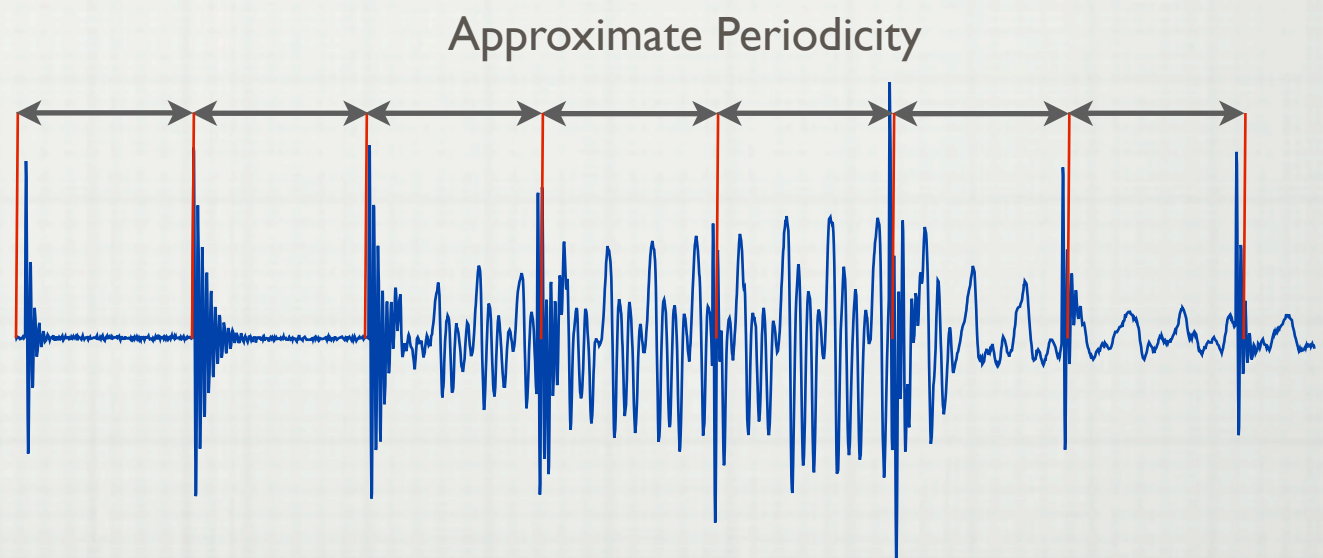
AUDIO FORENSICS APPLICATION: SINGLE CHANNEL SOURCE SEPARATION

- It is of interest to detect if the device is delivering current or not from a single mixture of the sources, referred to as **Single-Channel Source Separation**
- On the whole, **A Single Channel Semi-Blind Source Separation problem.**

Nominally Periodic signal $x_p \in \mathbb{R}^n$



Background Noise $x_u \in \mathbb{R}^n$



Single Linear Mixture

$x \in \mathbb{R}^n$

MOTIVATION

OUTLINE : BIRD'S EYE VIEW OF THE PRESENTATION

- ☐ Background: Setting the stage
- ☐ Semi-Blind Morphological Component Analysis (SBMCA)
- ☐ Evaluation of SBMCA : Simulation Specifics
- ☐ Conclusions and Future Work

BACKGROUND

MODEL

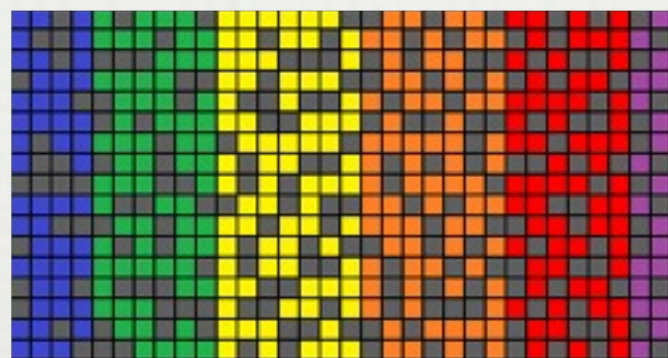
- We suppose that, m is an integer which divides n into q equal parts
- x is represented by matrix $X \in \mathbb{R}^{m \times q}$, and

$$X = X_p + X_u$$

- The aim is to separate X into its constituent matrices X_p and X_u

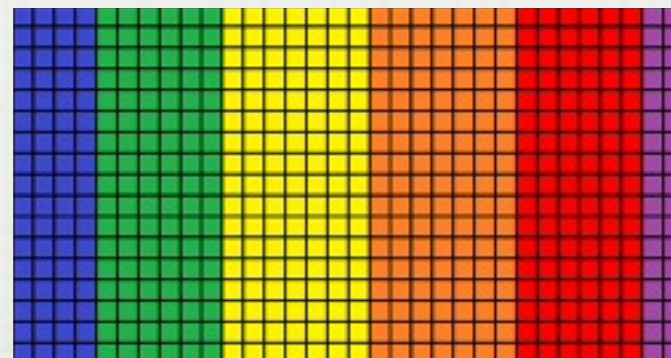
TRUNCATED-SINGULAR VALUE DECOMPOSITION(SVD)

- Let X_p be a rank-r matrix and X_u be random Gaussian noise.
- Estimating X_p is equivalent to finding a rank-r approximation of X



Noisy Data

=

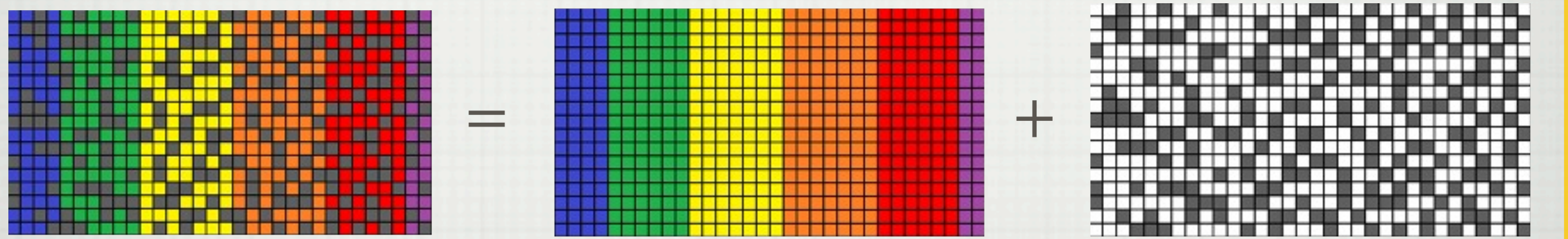


Low-Rank
Approximation

BACKGROUND

ROBUST PCA^[1,2]

- Let X_u be comprised of impulsive noise, in such a case we adopt Robust PCA, leading to following decomposition :



Data

Low-Rank

Sparse

BACKGROUND

LOW-RANK PLUS SPARSE IN A KNOWN DICTIONARY^[3]

- In case X_u is sparse in some known dictionary,

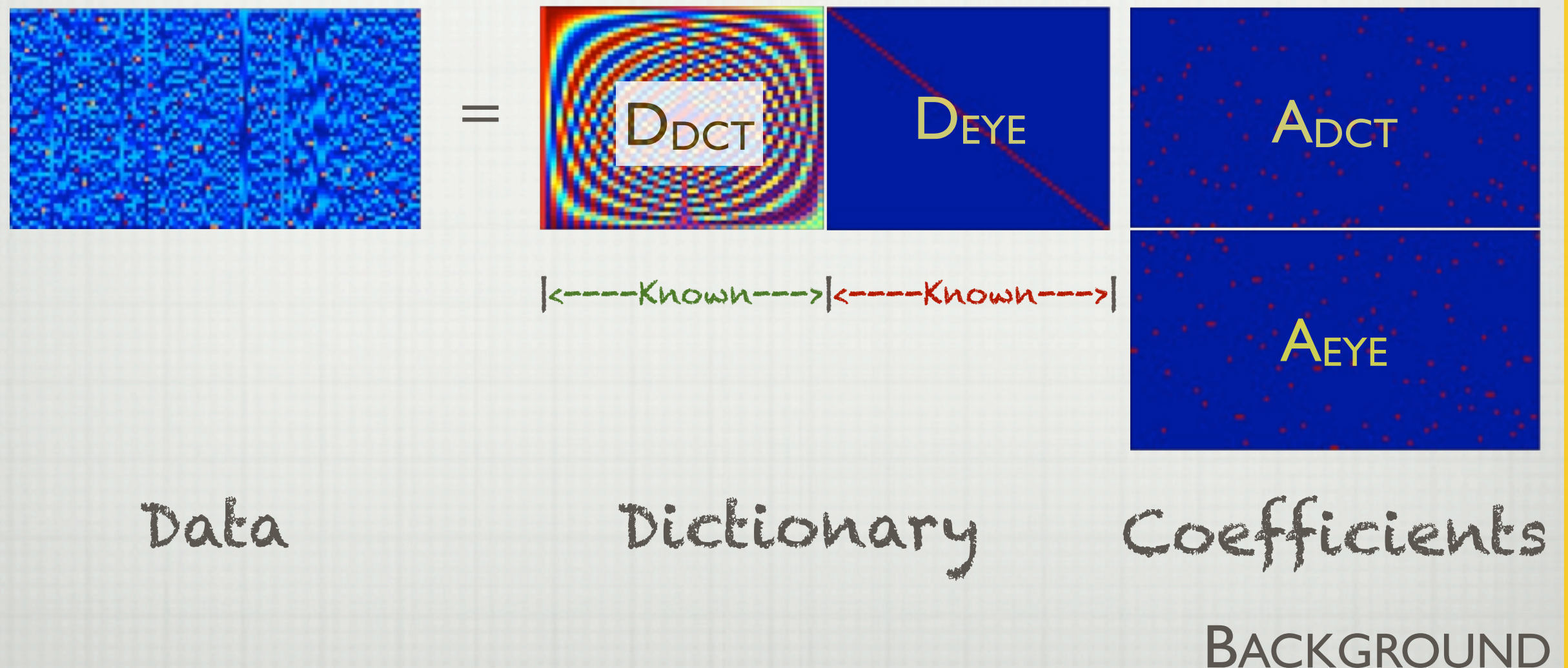
The diagram illustrates the Low-Rank Plus Sparse model in a known dictionary. It shows a large grid of colored squares (Data) equal to a large grid of colored squares (Low-Rank) plus a small grid of colored squares (Dictionary Sparse) multiplied by a small grid of black and white squares (Sparse).

Data = Low-Rank + Dictionary Sparse

BACKGROUND

MORPHOLOGICAL COMPONENT ANALYSIS^[4,5,6]

- Another extension: X_p and X_u are both sparse in some known dictionaries, represented as :



REVISITING THE CELLIST: So, How Do I Identify What Cello Sounds Like?



- ☐ I can listen to a sample of Cello before the next act.
- ☐ I **Train** my ears to Cello.
- ☐ Training data required.

Or

- ☐ I know how other instruments sound.
- ☐ I **Learn** the features of Cello by employing my prior **Experience** with other instruments.
- ☐ An **Online** methodology.

REVISITING THE CELLIST: HOW DO I IDENTIFY WHAT CELLO SOUNDS LIKE?



Motivation for our approach



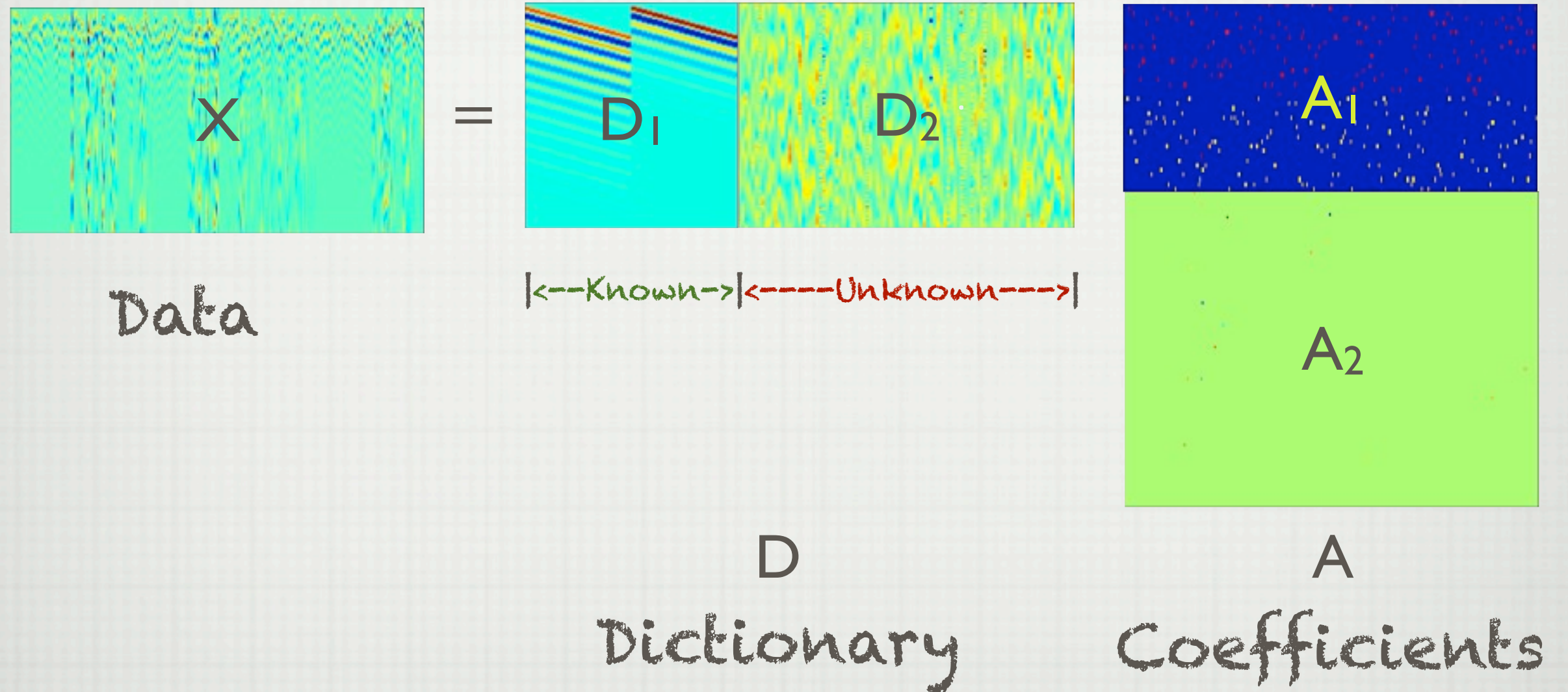
Or

- ☐ I can listen to a sample of Cello before the next act.
- ☐ I **Train** my ears to Cello.
- ☐ Training data required.

- ☐ I know how other instruments sound.
- ☐ I **Learn** the features of Cello by employing my prior **Experience** with other instruments.
- ☐ An **Online** methodology.

SEMI-BLIND MORPHOLOGICAL COMPONENT ANALYSIS (SBMCA)

SEMI-BLIND MORPHOLOGICAL COMPONENT ANALYSIS^[7]



SEMI-BLIND MORPHOLOGICAL COMPONENT ANALYSIS: ALGORITHMIC CONSIDERATIONS

- Formally we set out to solve

$$\{\hat{A}_1, \hat{A}_2, \hat{D}_2\} = \arg \min_{A_1, A_2, D_2} \|X - D_1 A_1 - D_2 A_2\|_F^2 + \tilde{\lambda}_1 \|A_1\|_1 + \tilde{\lambda}_2 \|A_2\|_1$$

for $\tilde{\lambda}_1, \tilde{\lambda}_2 > 0$

- This optimization problem is

- Not jointly convex

- Sensitive to initialization

- We adopt

- Alternating Minimization based approach for Online Dictionary Learning^[8,9,10]

SBMCA

SEMI-BLIND MORPHOLOGICAL COMPONENT ANALYSIS

ALGORITHMIC DETAILS

Algorithm 1: Semi-Blind MCA Algorithm

Input: Original Data $X \in \mathbb{R}^{m \times q}$, Known Dictionary $D_1 \in \mathbb{R}^{m \times d}$,
Regularization parameters $\lambda_1, \lambda_2, \lambda_3 > 0$,
Number of elements in unknown dictionary ℓ .

Initialize: $\tilde{A}_1 \leftarrow \arg \min_{A_1} \|X - D_1 A_1\|_F^2 + \lambda_1 \|A_1\|_1$
(or other suitable initialization depending on the problem.)

Iterate (repeat until convergence):

repeat

Dictionary Learning:

$$\{\tilde{D}_2, \tilde{A}_2\} \leftarrow \arg \min_{D_2, A_2} \|X - D_1 \tilde{A}_1 - D_2 A_2\|_F^2 + \lambda_2 \|A_2\|_1$$

Coefficient Update:

$$\begin{aligned} \tilde{D} &= [D_1 \ \tilde{D}_2] \\ [\tilde{A}_1^T \ \tilde{A}_2^T]^T &\triangleq \tilde{A} \leftarrow \arg \min_A \|X - \tilde{D} A\|_F^2 + \lambda_3 \|A\|_1 \end{aligned}$$

until convergence

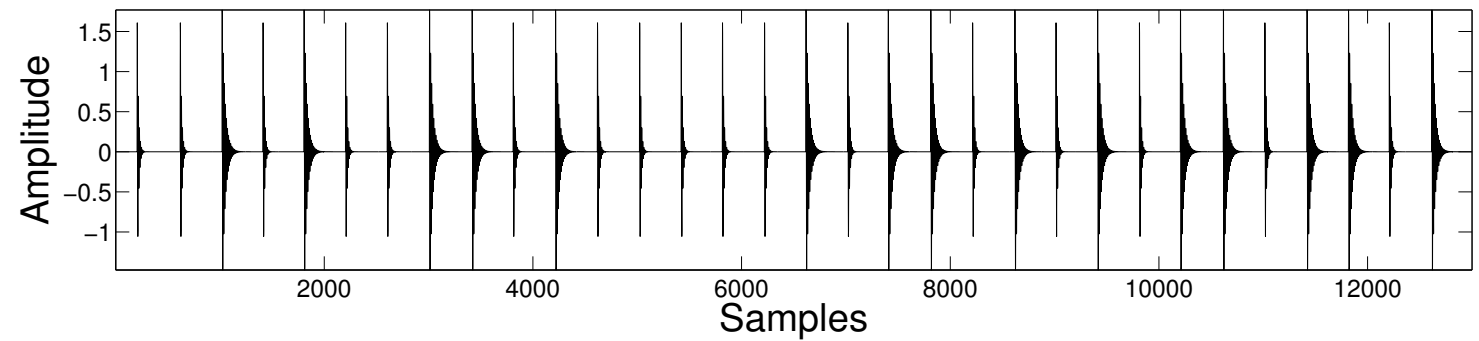
Output: Learned dictionary $\hat{D}_2 \leftarrow \tilde{D}_2$,
Coefficient estimates $\hat{A}_1 = \tilde{A}_1$, $\hat{A}_2 = \tilde{A}_2$.

SBMCA

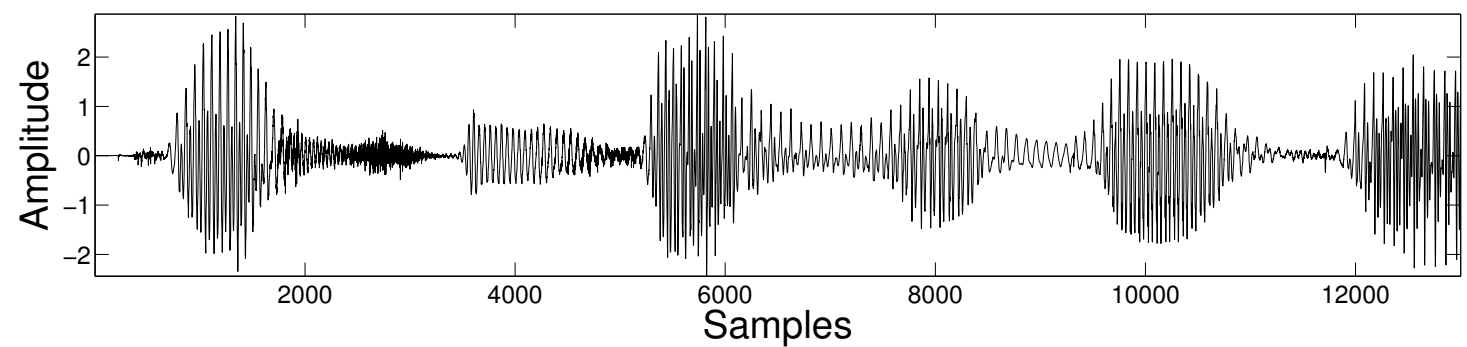
EVALUATION OF PERFORMANCE

SIGNAL CONFIGURATION

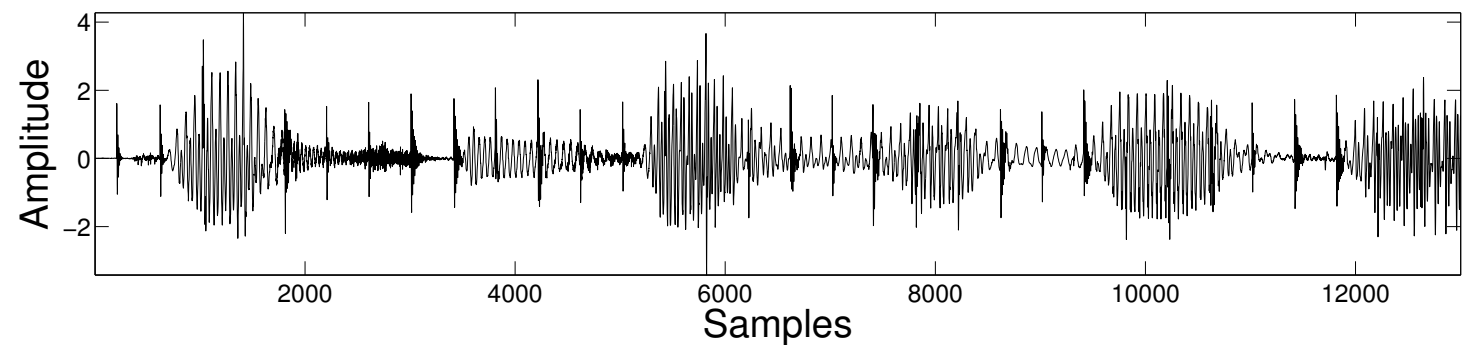
Nominally Periodic Signal



Unknown Background Signal



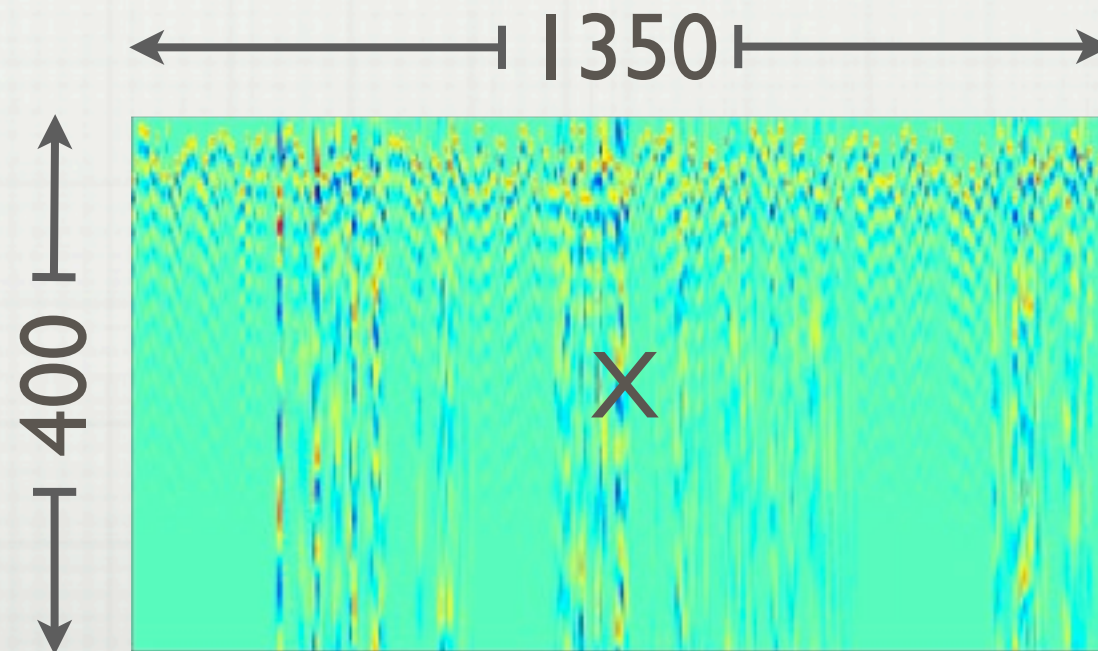
Linear Mixture



EVALUATION

DATA GENERATION







- Data formed from a mixture of Speech¹ (unknown) and nominally periodic signal (one per period)
- Data matrix looks like,



¹ Speech Samples obtained from VoxForge Speech Corpus: www.voxforge.org/home

EVALUATION

FREQUENCY DOMAIN SOURCE SEPARATION

Method	D_1	D_2
SBMCA		 Unknown
MCA-DCT-Fourier		 $ \mathcal{F}\{\text{DCT}\} $
MCA-Identity-Fourier		 $ \mathcal{F}\{\text{Identity}\} $

SOURCE SEPARATION IN FREQUENCY DOMAIN

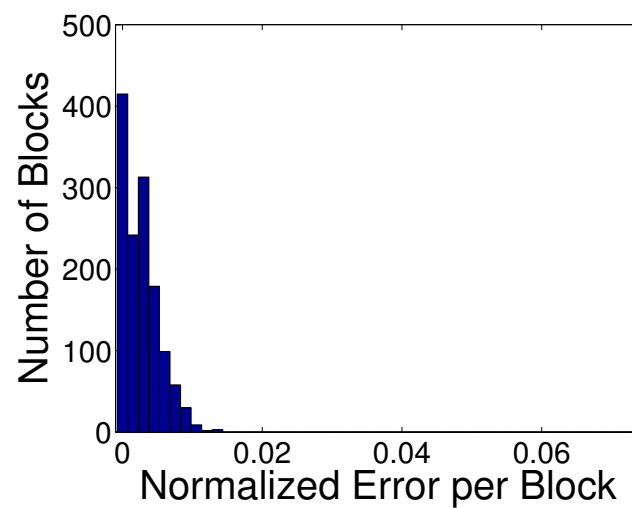
Table 1: Analysis of reconstruction SNR(in dB): Frequency Domain Separation

Noise $\mathcal{N}(0, \sigma^2)$	$\sigma = 0$		$\sigma = 0.001$		$\sigma = 0.01$		$\sigma = 0.1$	
Method \ Signal	x_p	x_u	x_p	x_u	x_p	x_u	x_p	x_u
SBMCA	8.95	15.21	8.91	15.17	8.83	15.09	6.80	11.56
MCA-DCT-Fourier	8.81	15.16	8.81	15.16	8.88	15.19	6.82	12.50
MCA-Identity-Fourier	1.19	-19.07	1.19	-19.07	1.19	-18.90	1.34	-9.57

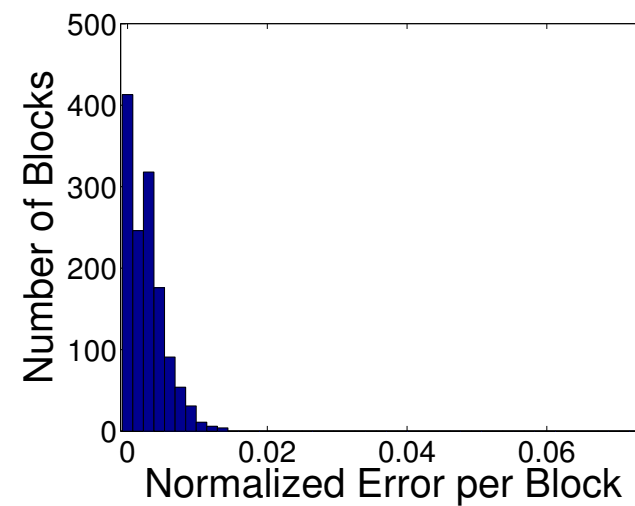
EVALUATION

SOURCE SEPARATION IN FREQUENCY DOMAIN: NOMINALLY PERIODIC SIGNAL

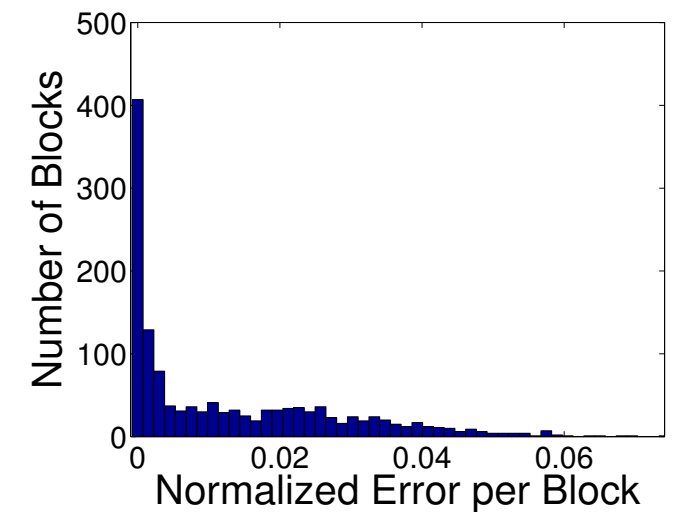
SBMCA



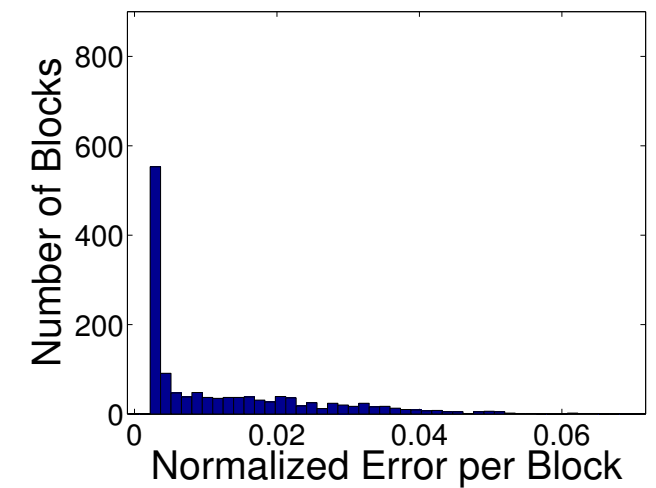
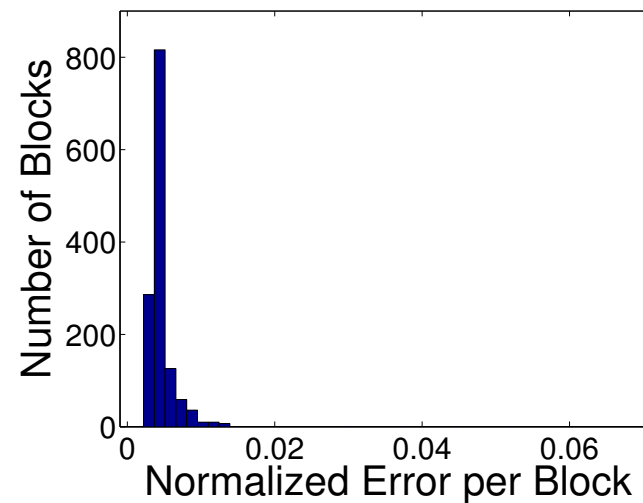
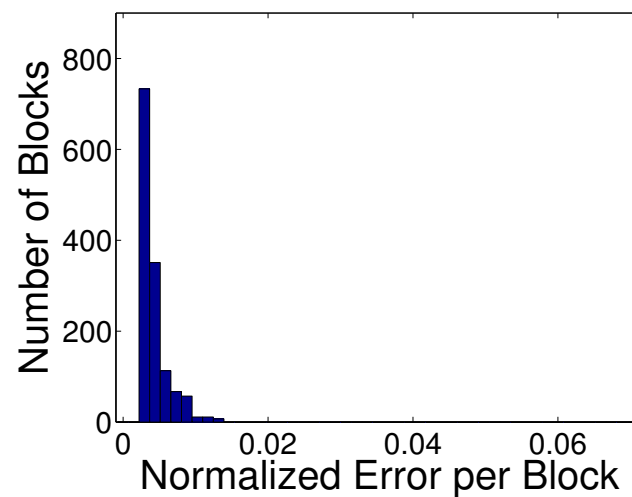
MCA-DCT-
FOURIER



MCA-Identity-
Fourier



$\sigma = 0.1$

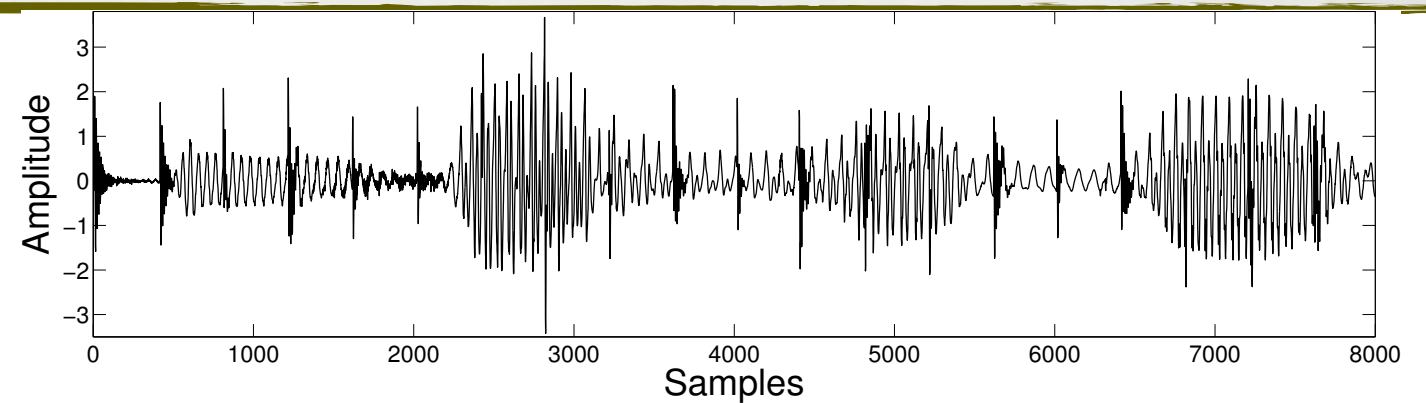


EVALUATION

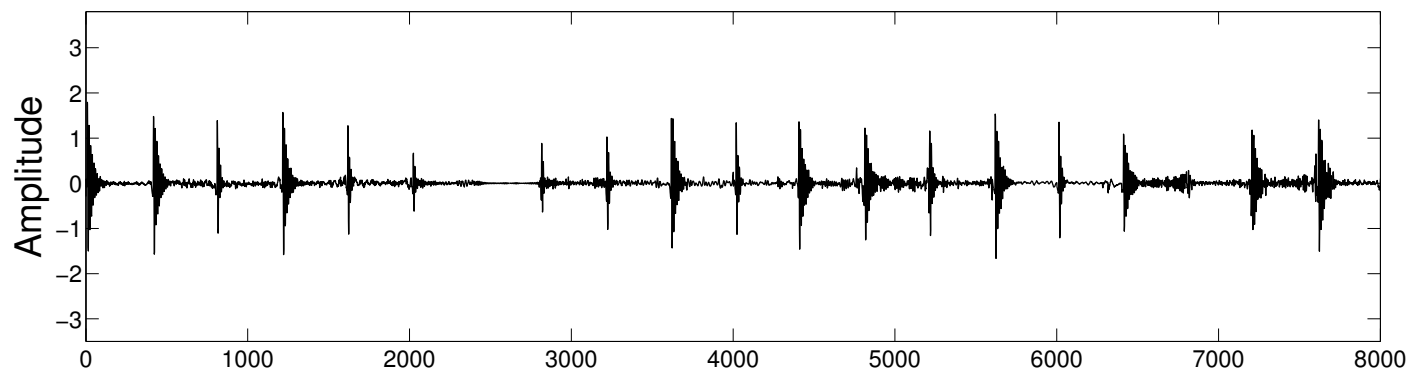
SOURCE SEPARATION IN FREQUENCY DOMAIN: NOMINALLY PERIODIC SIGNAL

$$\sigma = 0$$

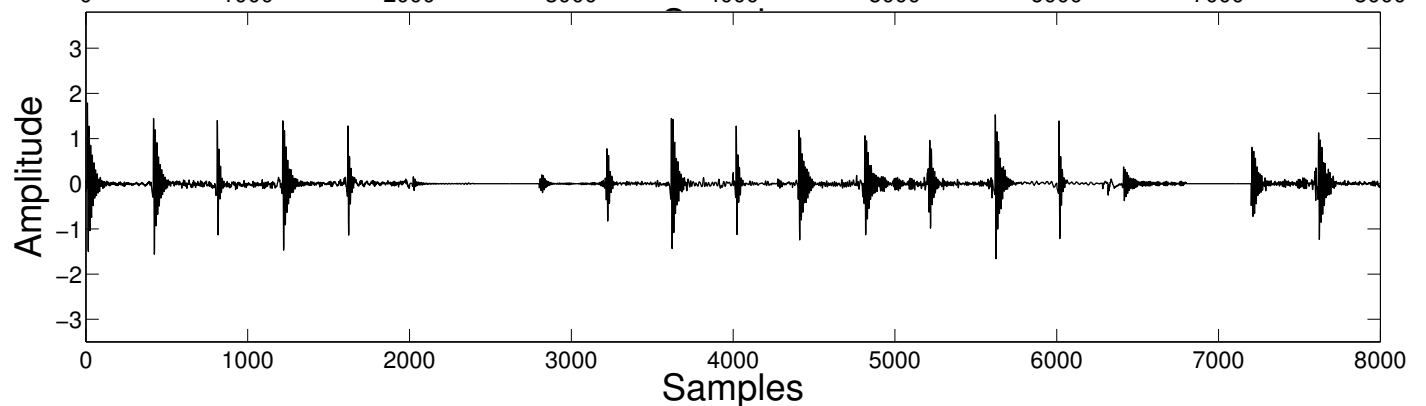
Original Mixture



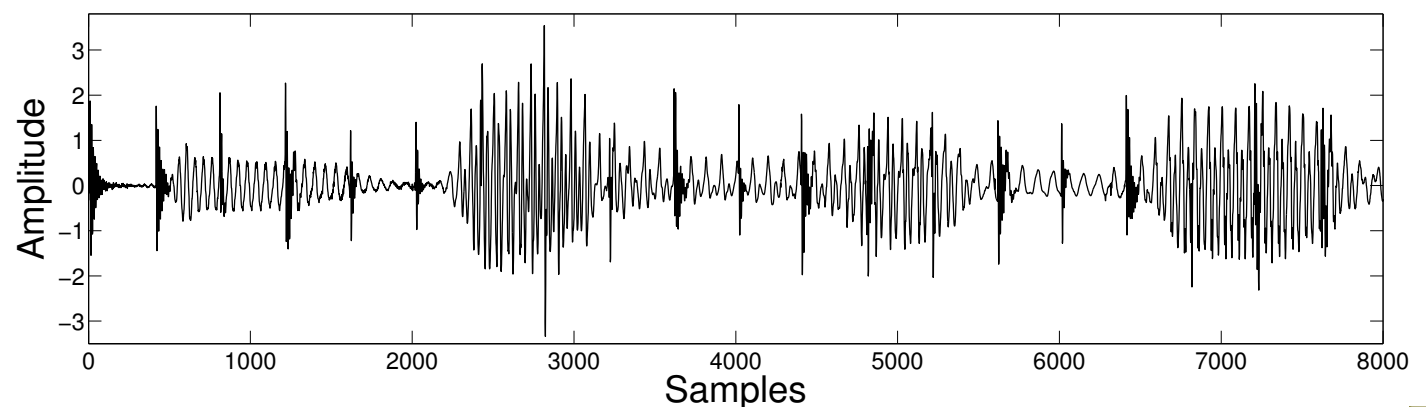
SBMCA



MCA-DCT-Fourier



MCA-Identity-Fourier



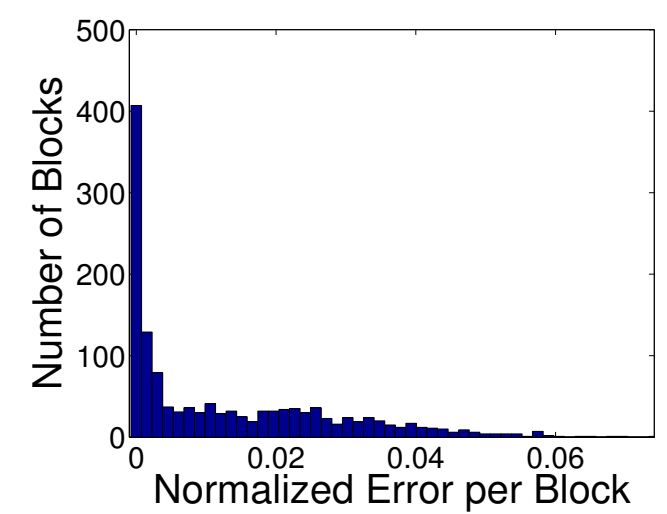
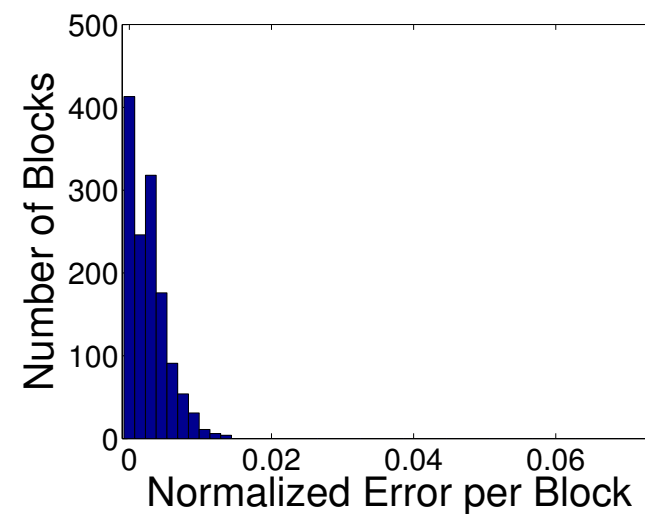
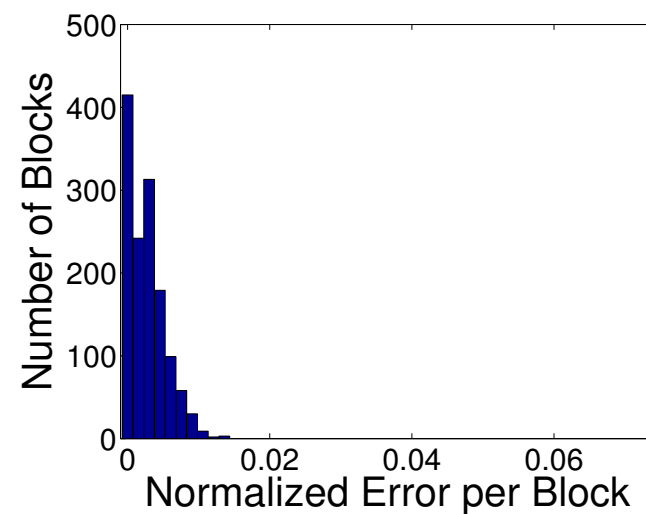
SOURCE SEPARATION IN FREQUENCY DOMAIN: BACKGROUND SIGNAL

SBMCA

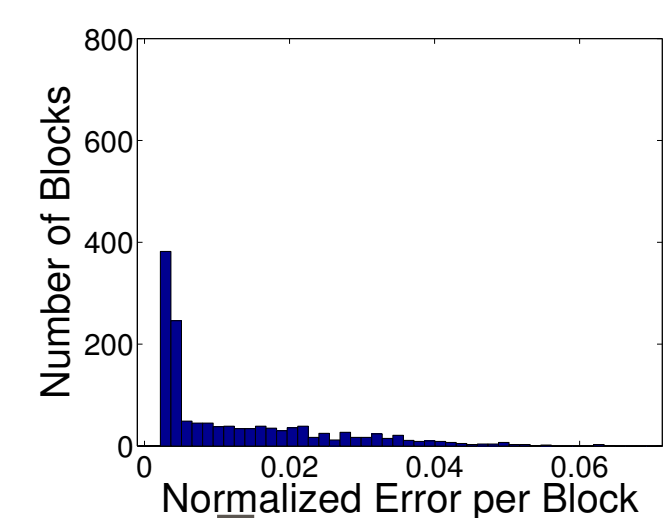
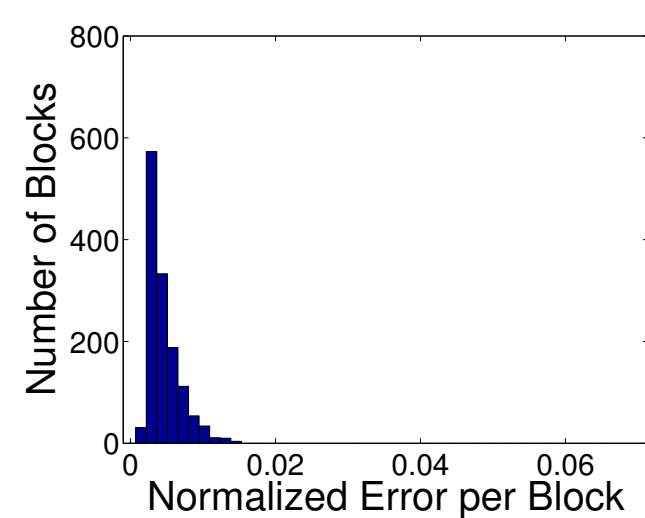
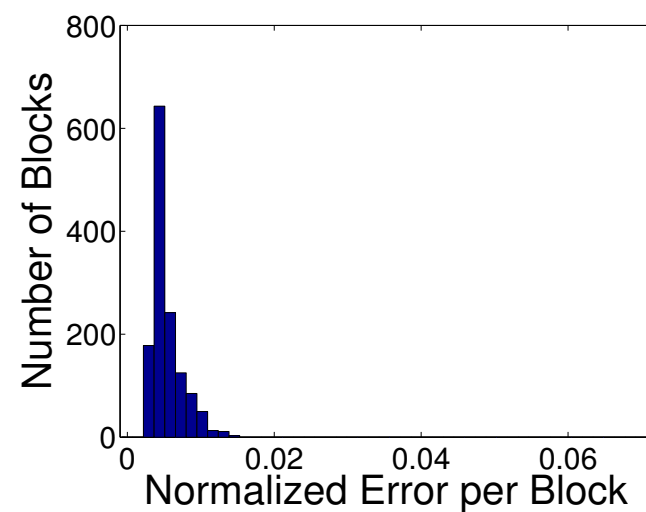
MCA-DCT-
FOURIER

MCA-Identity-
Fourier

$\sigma = 0$



$\sigma = 0.1$

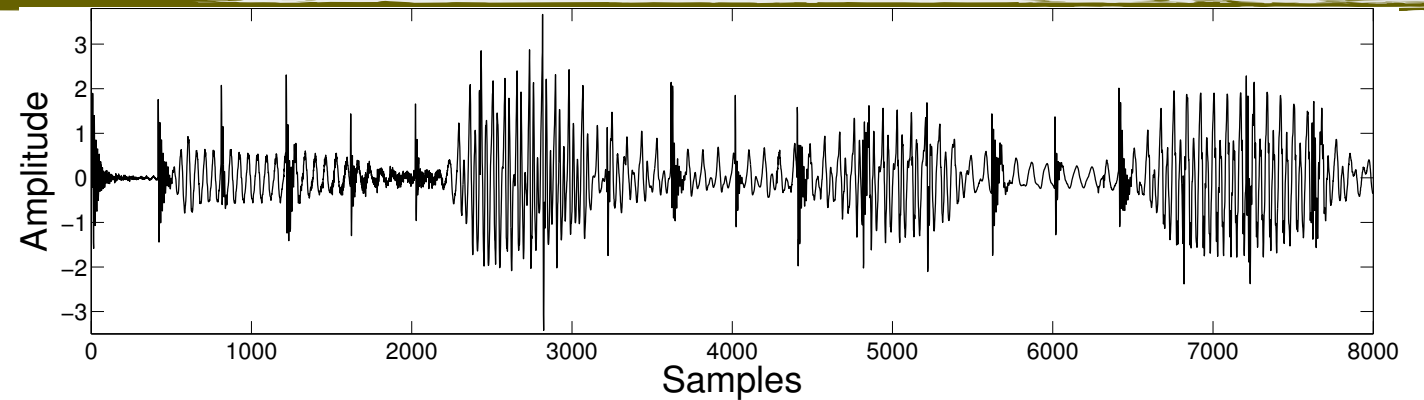


EVALUATION

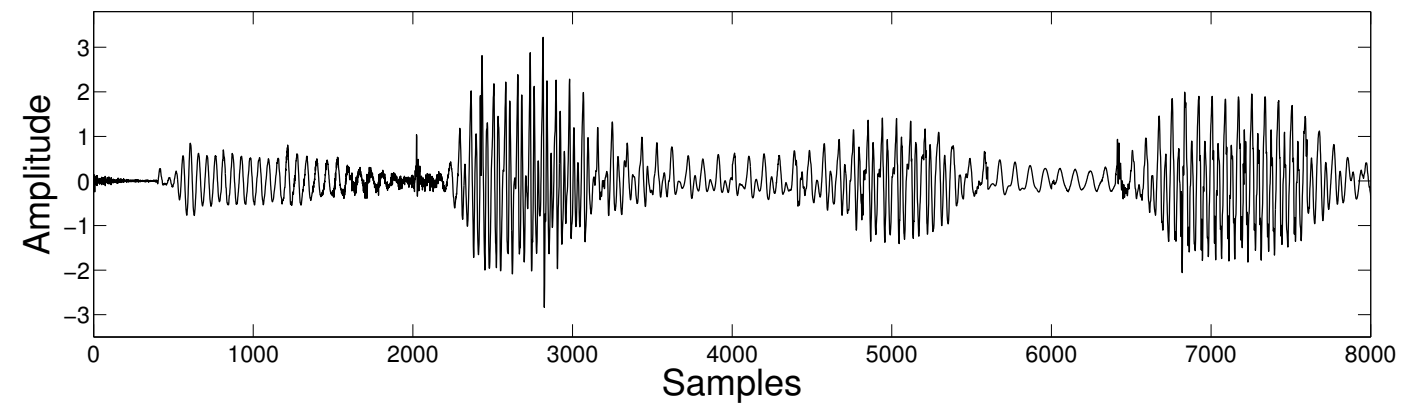
SOURCE SEPARATION IN FREQUENCY DOMAIN: BACKGROUND SIGNAL

$$\sigma = 0$$

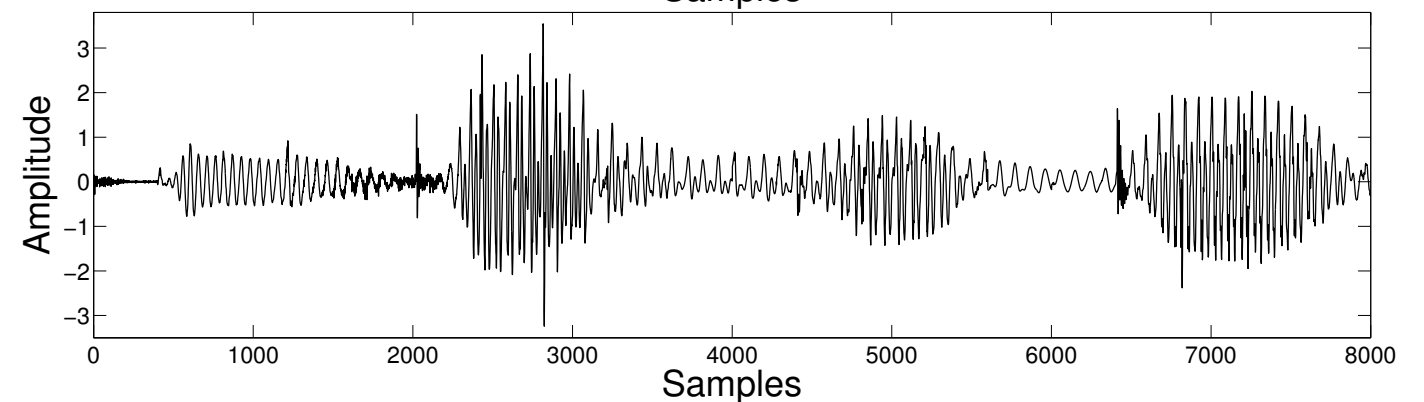
Original Mixture



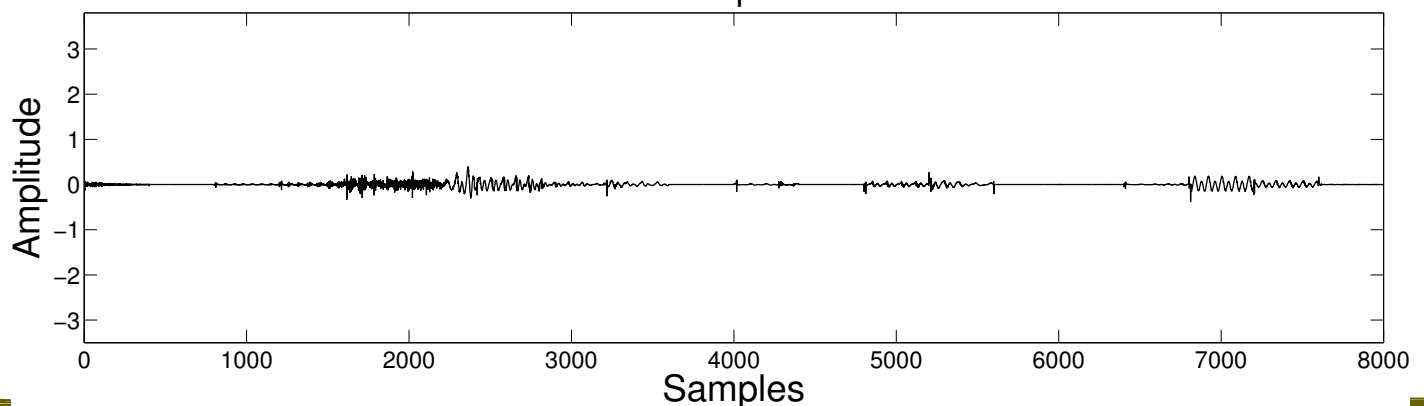
SBMCA




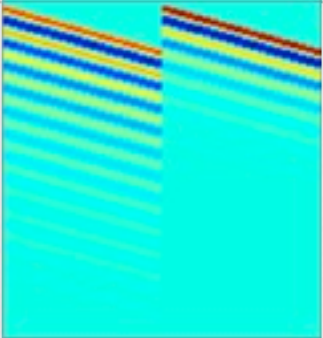

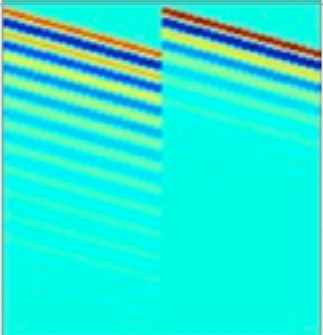
MCA-DCT-Fourier



MCA-Identity-Fourier



TIME DOMAIN SOURCE SEPARATION

Method	D_1	D_2
SBMCA		 Unknown
MCA-DCT		 DCT
MCA-Identity		 Identity

SOURCE SEPARATION IN TIME DOMAIN

Table 2: Analysis of reconstruction SNR(in dB): Time Domain Separation

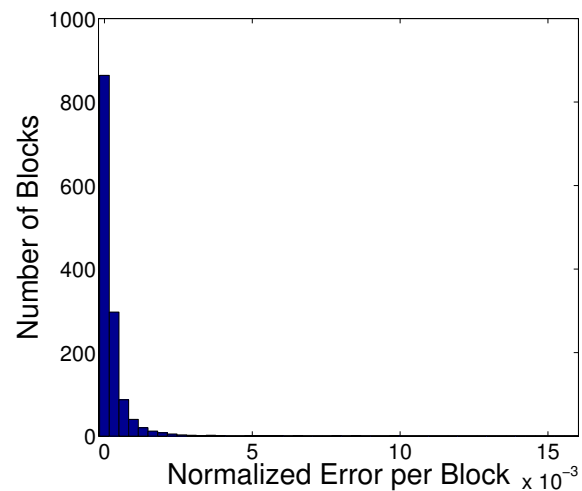
Noise $\mathcal{N}(0, \sigma^2)$	$\sigma = 0$		$\sigma = 0.001$		$\sigma = 0.01$		$\sigma = 0.1$	
Method \ Signal	x_p	x_u	x_p	x_u	x_p	x_u	x_p	x_u
SBMCA	23.72	29.32	23.73	29.32	23.08	27.40	19.72	16.84
MCA-DCT	20.44	26.02	20.46	26.02	20.19	24.96	18.09	16.72
MCA-Identity	10.90	16.06	10.90	16.44	10.90	16.33	10.78	11.44

EVALUATION

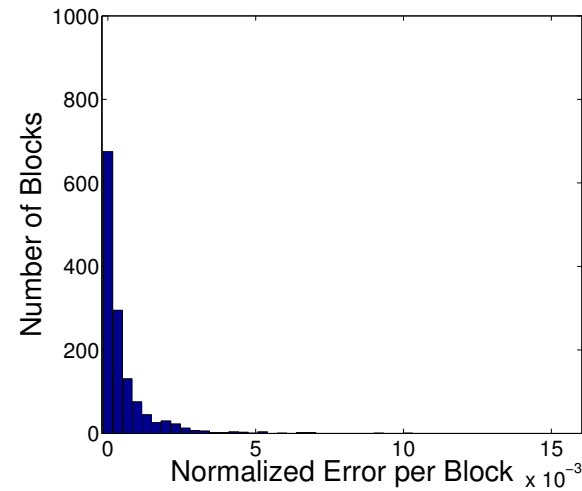
SOURCE SEPARATION IN TIME DOMAIN: NOMINALLY PERIODIC SIGNAL

$$\sigma = 0$$

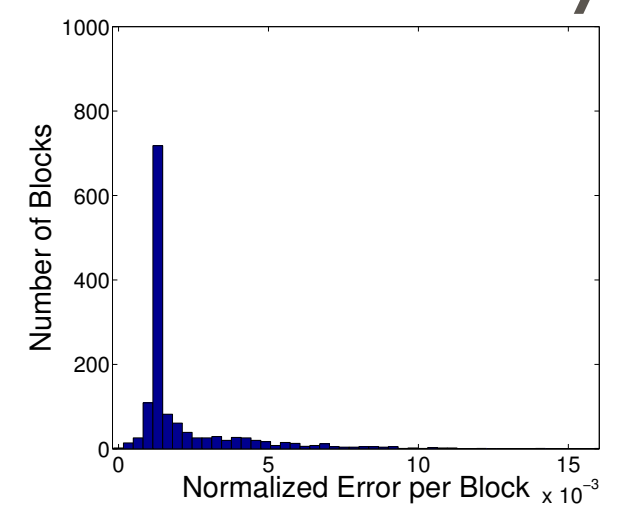
SBMCA



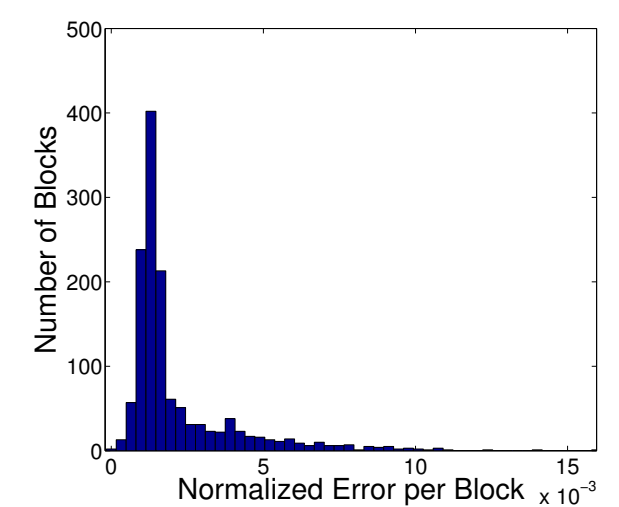
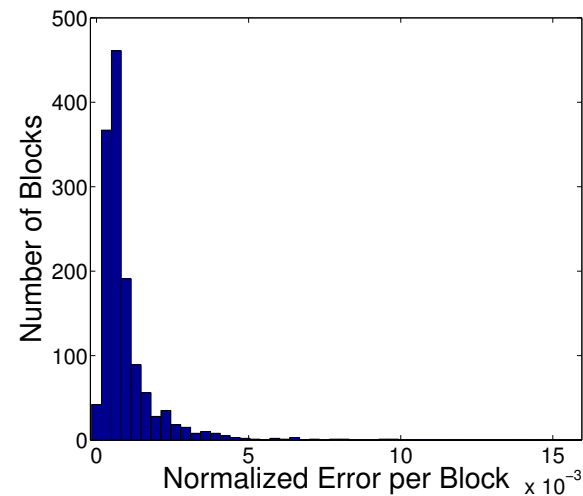
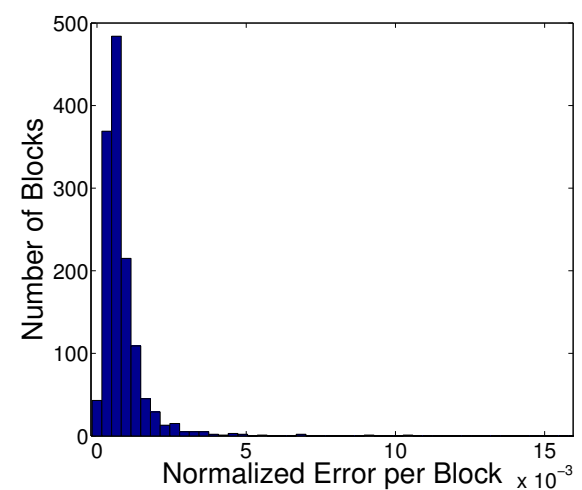
MCA-DCT



MCA-Identity



$$\sigma = 0.1$$

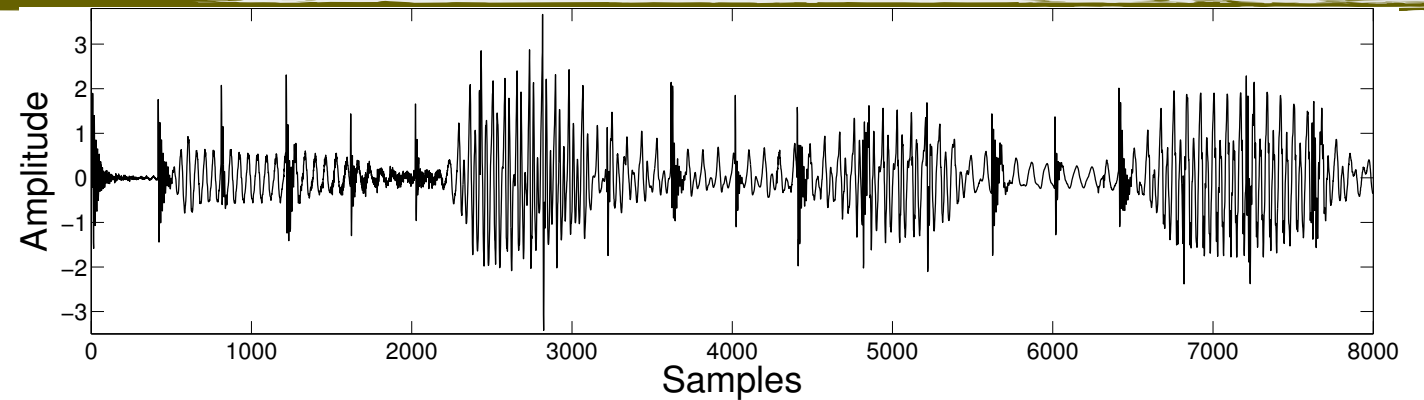


EVALUATION

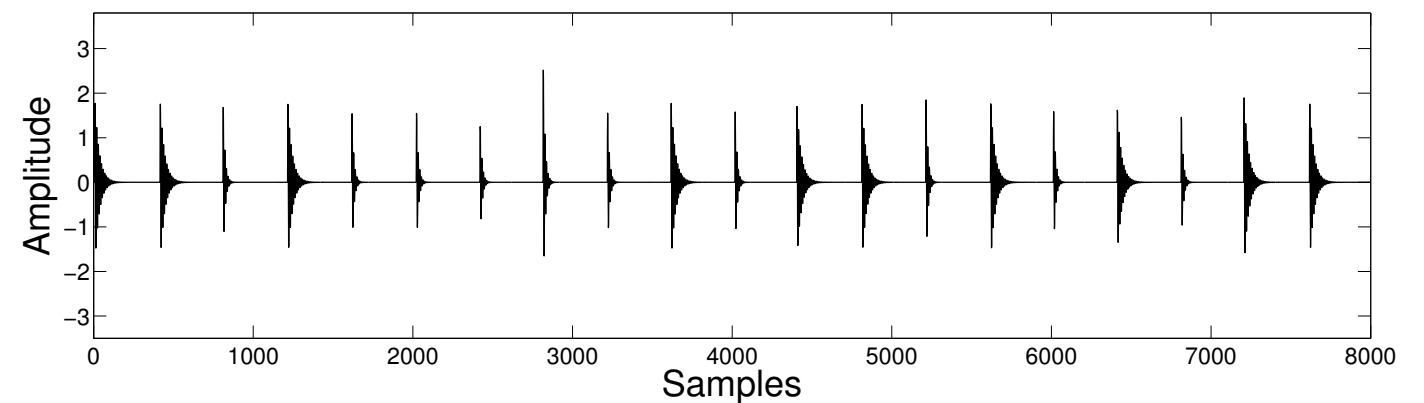
SOURCE SEPARATION IN TIME DOMAIN: NOMINALLY PERIODIC SIGNAL

$$\sigma = 0$$

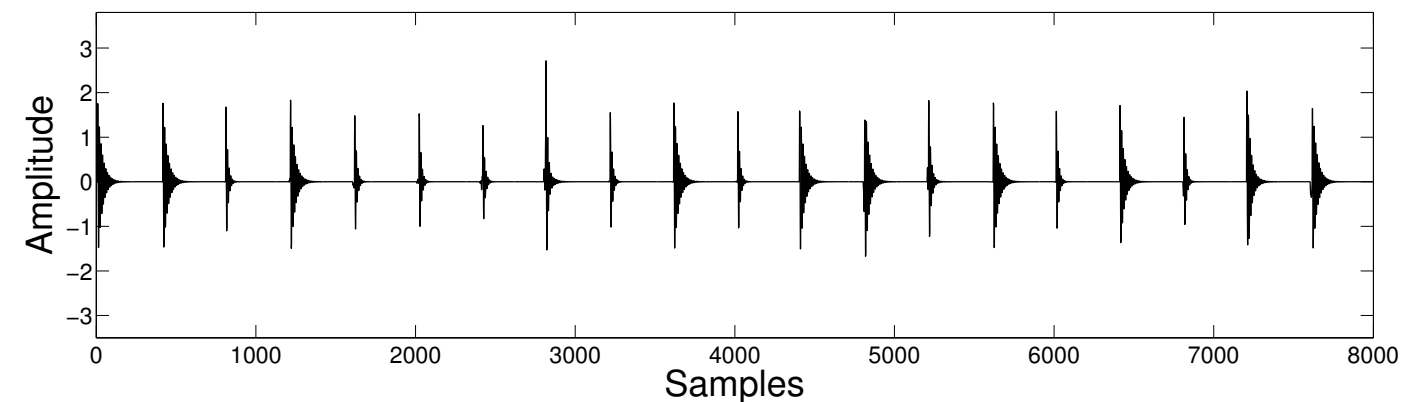
Original Mixture



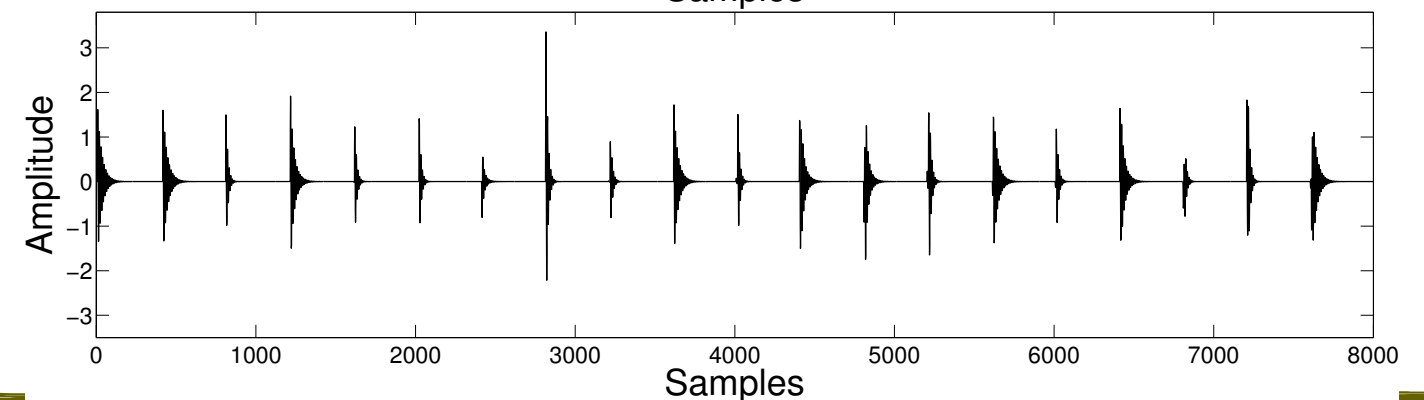
SBMCA



MCA-DCT

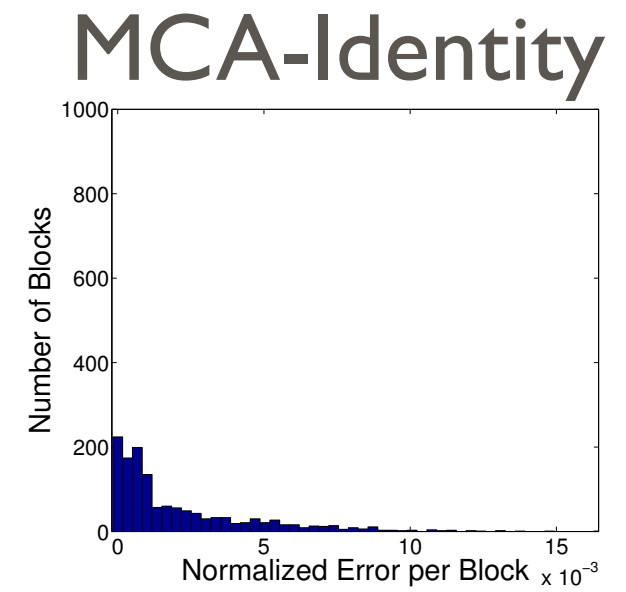
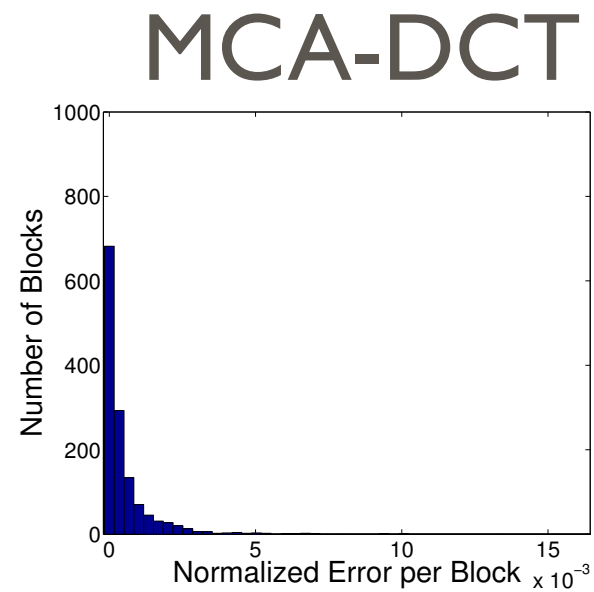
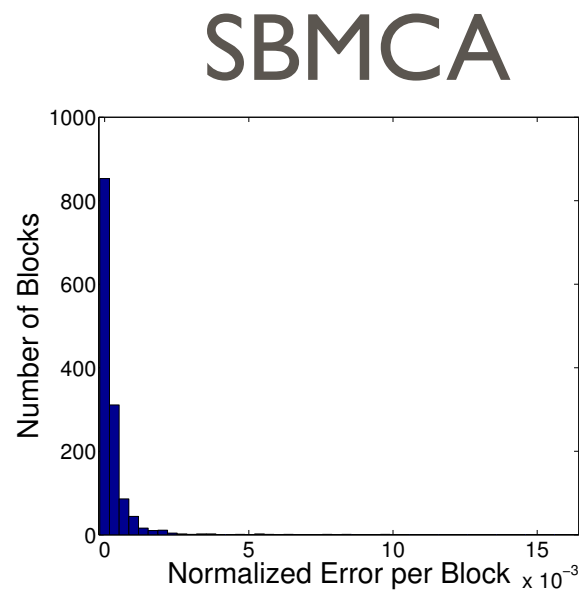


MCA-Identity

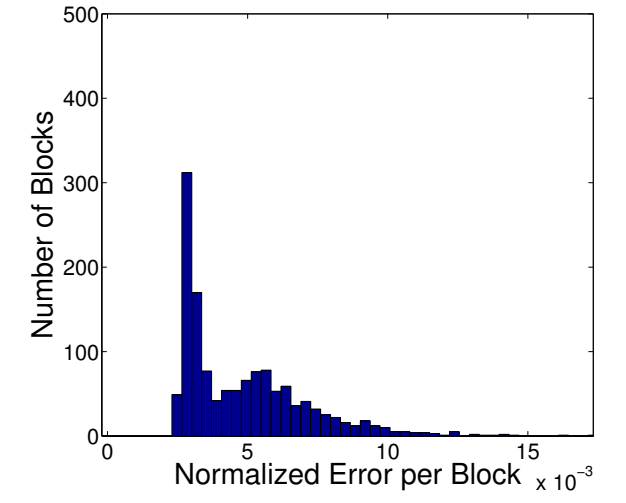
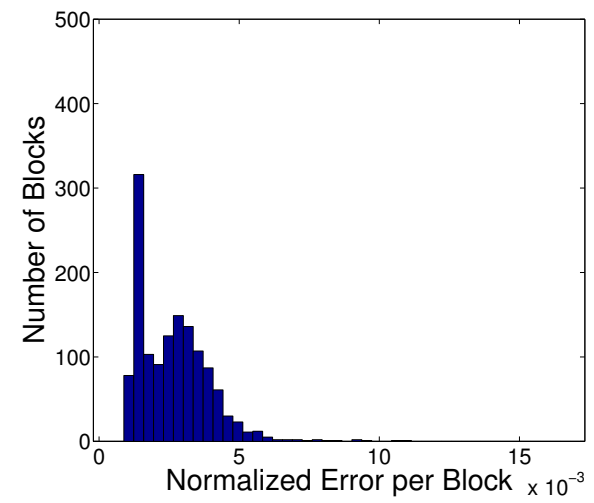
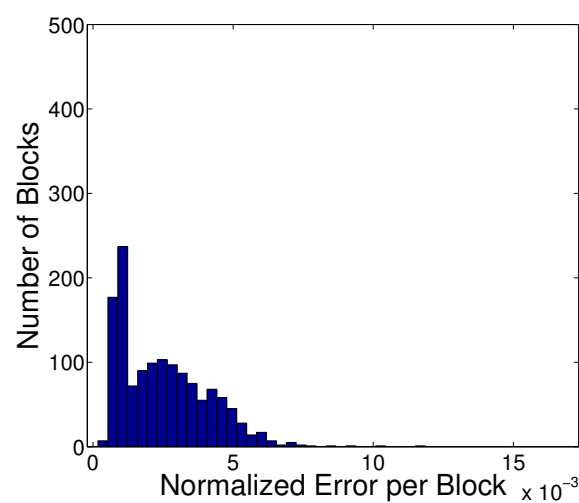


SOURCE SEPARATION IN TIME DOMAIN: BACKGROUND SIGNAL

$\sigma = 0$



$\sigma = 0.1$

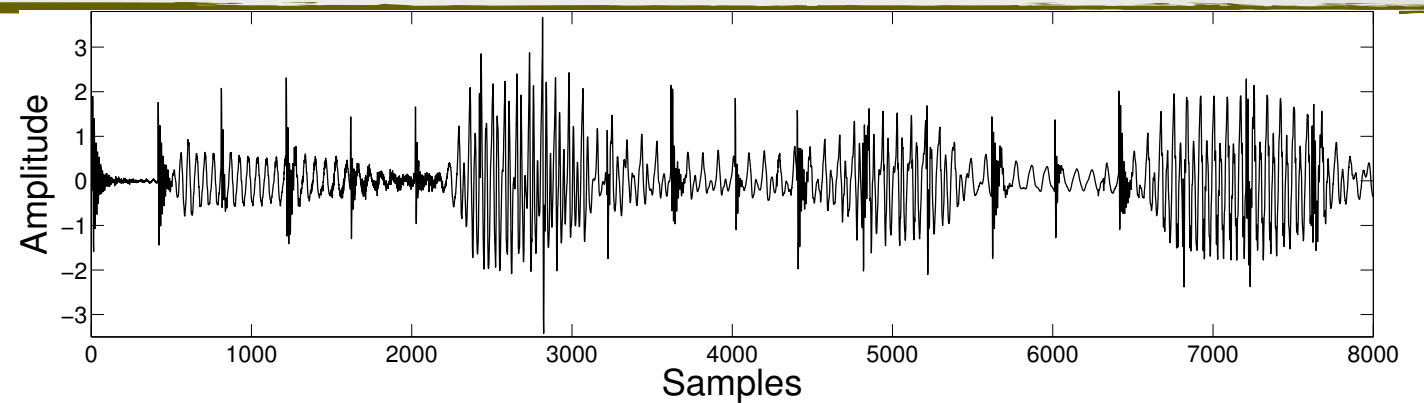


EVALUATION

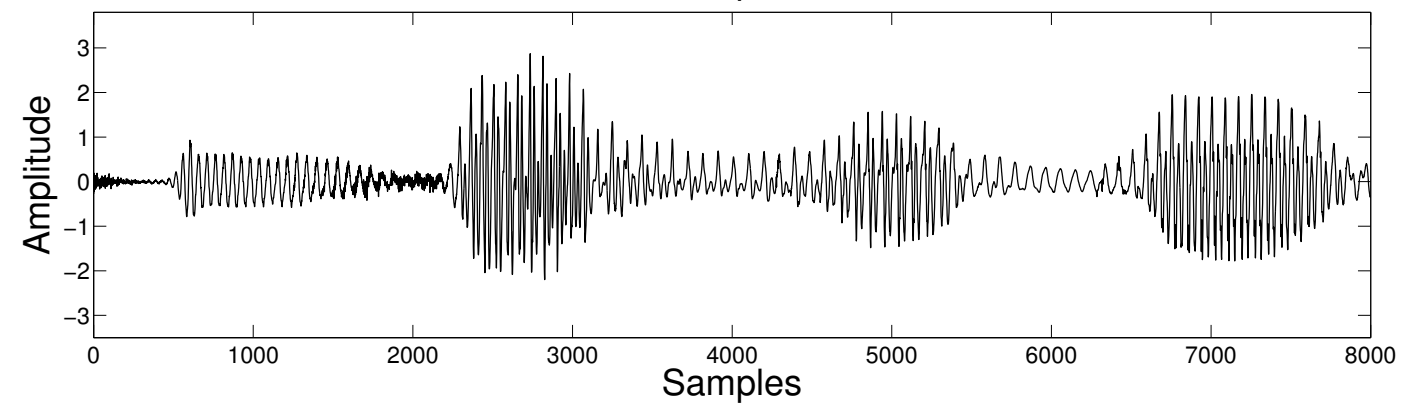
SOURCE SEPARATION IN TIME DOMAIN: BACKGROUND SIGNAL

$$\sigma = 0$$

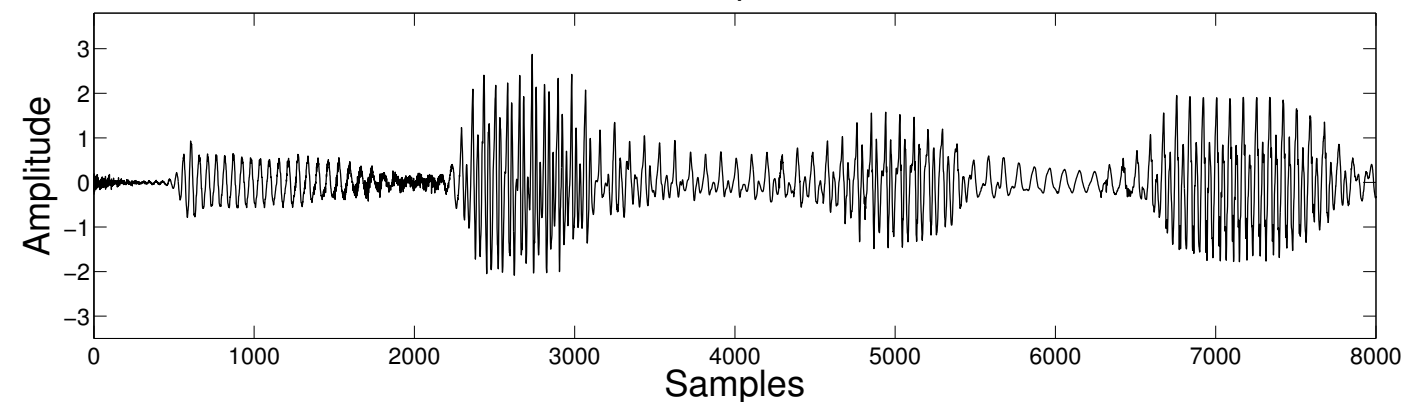
Original Mixture



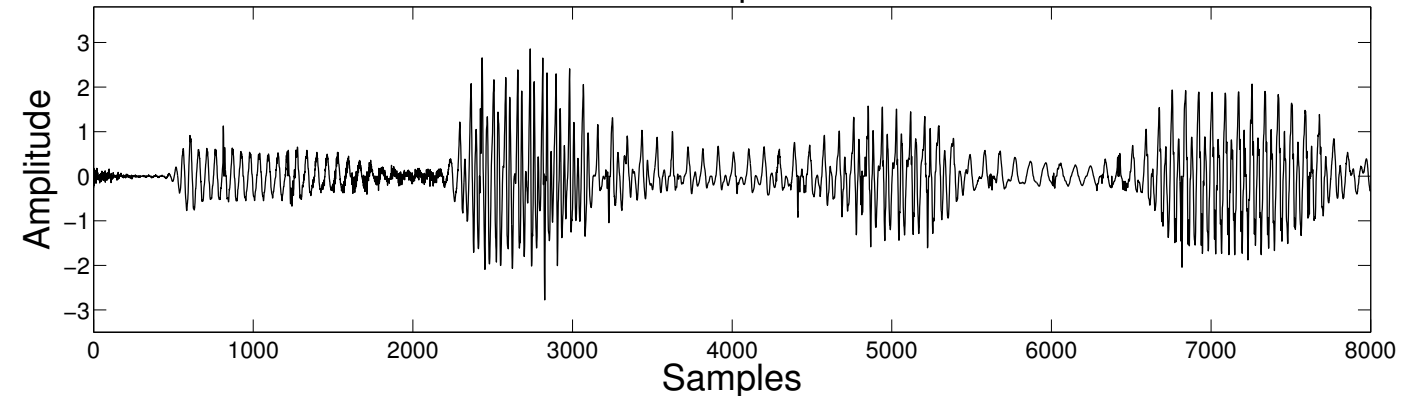
SBMCA



MCA-DCT



MCA-Identity



CONCLUSIONS

- ☐ Our approach exploits partial prior knowledge of one of the sources, in the form of a dictionary which sparsely represents local segments of one of the sources. A key feature being online learning of a dictionary (from the mixed source data itself) for representing the unknown background source.
- ☐ The timing uncertainty inherent in our application suggests that our approach may be combined with other existing alignment techniques [11, 12, 13].
- ☐ More recently [14] proposed a robust alignment procedure that can be viewed as an extension of robust PCA.
- ☐ We defer these extensions, as well as the investigation of our approach to other applications (e.g., in image or video processing) to future efforts.



REFERENCES

1. E. J. Candès, X. Li, Y. Ma, and J. Wright. Robust Principal Component Analysis? *Journal of the ACM*, 58(3):11, 2011.
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THANK YOU!

